Sensitivity analysis of STREAM burn-through simulations on TCV: identifying the dominant input parameters

F. Gâche[1], P. A. Molina-Cabrera[1], M. Hoppe[2], I. Ekmark[3], S. Coda[1], L. Simons[1], U. Sheikh[1], J. Decker[1], B. Vincent[1], and the TCV team

[1] *SPC-EPFL*, [2] *KTH*, [3] *Chalmers*

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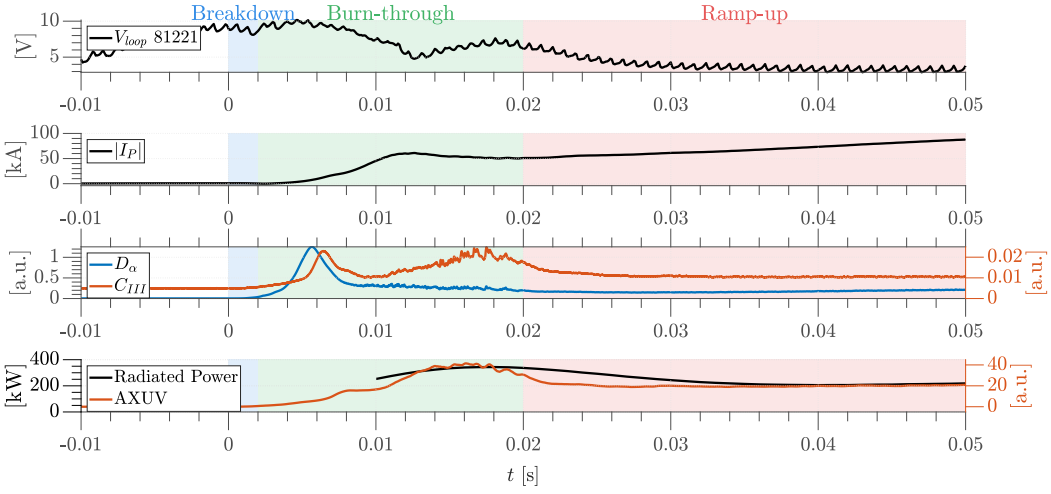
What is the start-up?



Runaway electrons in the burn-through

- REs: low electron density and/or a strong electric field
- REs are hard to control, can limit ohmic heating, and may damage the vessel
- ITER: $E_{\max} = 0.3\,\mathrm{V\,m^{-1}}$, $p_{\mathrm{prefill}}^{\max} = 2\,\mathrm{mPa}$ (de Vries & Gribov, Nucl. Fusion, 59, 2019)

STREAM

- STart-up Runaway Electron Analysis Model (Hoppe *et al.*, J. Plasma Phys., 88, 2022)
- 0D burn-through code.
  - Volume-averaged outputs
  - Fluid model
  - Self consistent runaway electron evolution

STREAM — equations

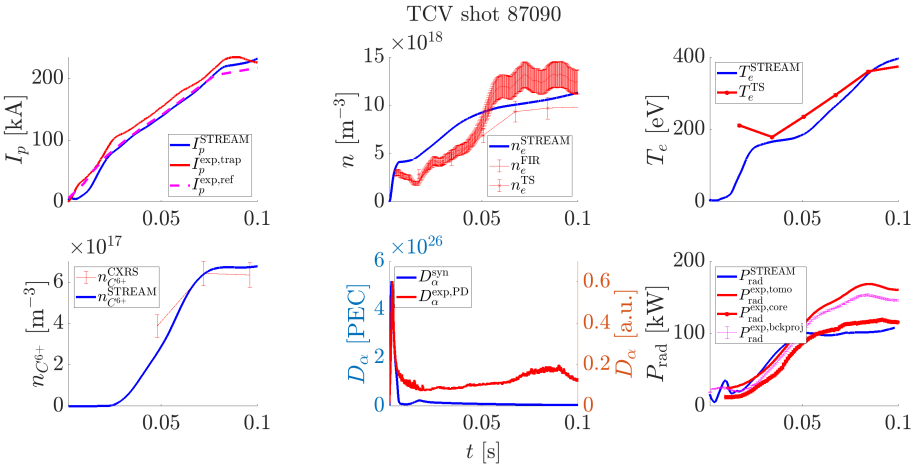
- Energy balance:
  - $\frac{\mathrm{d}W_e}{\mathrm{d}t} = j_\Omega E_\parallel - P_{\text{inelastic losses}} - \sum_i Q_{ei} - \frac{W_e}{\tau_e}$
  - $\frac{\mathrm{d}W_i}{\mathrm{d}t} = \sum_k Q_{ik} - P_{\text{losses}} - \frac{W_i}{\tau_i}$
- Particle balance: accounts for all species
- Circuit:
  - $2\pi R_0 E_\parallel + L_p \frac{\mathrm{d}I_p}{\mathrm{d}t} + M_{vp} \frac{\mathrm{d}I_{wall}}{\mathrm{d}t} = V_{loop}$
  - $R_{wall} I_{wall} + L_{wall} \frac{\mathrm{d}I_{wall}}{\mathrm{d}t} + M_{vp} \frac{\mathrm{d}I_p}{\mathrm{d}t} = V_{loop}$
- Runaway electron evolution: $\frac{\mathrm{d}n_{re}}{\mathrm{d}t} = \gamma_{Dreicer} + \Gamma_{ava} n_{re} - \frac{n_{re}}{\tau_{re}}$

STREAM — schematics

STREAM — Input parameters

- Physical features of TCV: $V_{vessel},\ R_{wall},\ L_{wall}, \ldots$
- Discharge parameters: $V_{loop},\ V_p,\ I_{wall}, \ldots$
- Uncontrolled parameters: $Y_a^b,\ T_{e,0},\ T_{i,0},\ n_{C,0}, \ldots$

STREAM — Outputs



Sensitivity analysis — the question

Which input parameters most influence burn-through dynamics and runaway-electron generation?

Sensitivity workflow

1. sample perturbed STREAM inputs,
2. compare each run with a reference run,
3. reduce all output variation to one score ϖ,
4. rank inputs by the induced variation of ϖ.

Uncertain inputs

STREAM: $K \approx 38$ inputs. For each input,

$$x_k \in [\mu_k - \sigma_k,\ \mu_k + \sigma_k].$$

μ_k: the average
σ_k: the uncertainty.

Step 1 — score the variation to the reference

Each run produces time traces. For one monitored quantity Y, compare it with the reference trace, point by point:

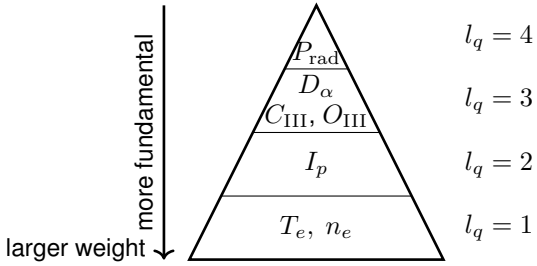
Raw variation

$$\mathrm{RMSE}(Y, Y^{ref}) = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\left(Y_m - Y_m^{ref}\right)^2}$$

Dimensionless variation

$$\mathrm{NRMSE}(Y, Y^{ref}) = \frac{\mathrm{RMSE}(Y, Y^{ref})}{\mathrm{RMS}(Y^{ref})}$$

Step 2 — weight the outputs by importance



Weights:

$$w_q = \frac{1}{l_q}.$$

One discrepancy score per run:

$$\varpi = \frac{\sum_q w_q \ \mathrm{NRMSE}(Y_q, Y_q^{ref})}{\sum_q w_q}.$$

Large $\varpi \implies$ strong deviation on fundamental outputs.

Step 3a — the idea: companion runs that share one input

For each input we want *one number*: how strongly does it control the match score ϖ? (*small ϖ = good match, large = poor match*).

Run

	$T_{e,0}$ [eV]	$T_{i,0}$ [eV]
run 1	1.0	0.4
run 2	1.5	0.6
run 3	2.8	0.3

one row = one full run $\rightarrow \varpi_b$

Companion

	$T_{e,0}$ [eV]	$T_{i,0}$ [eV]
run 1	1.0	0.5
run 2	1.5	0.3
run 3	2.8	0.7

same $T_{e,0}$, new $T_{i,0} \rightarrow \varpi_b^{(k)}$

A run and its companion share **only** $T_{e,0}$.

Step 3b — the sample matrices

One companion is noisy, so repeat it B times. Each row e_b of $E \in \mathbb{R}^{B\times K}$ is one configuration; **one STREAM run per row**. For parameter k: keep the blue column, redraw the rest ($\curvearrowright$) $\Rightarrow E^{(k)}$.

$$\underbrace{\begin{bmatrix} e_{1,1} & \cdots & e_{1,k} & \cdots & e_{1,K} \\ \vdots & & \vdots & & \vdots \\ e_{B,1} & \cdots & e_{B,k} & \cdots & e_{B,K} \end{bmatrix}}_{E} \rightarrow \begin{matrix} \varpi_1 \\ \vdots \\ \varpi_B \end{matrix} \qquad \underbrace{\begin{bmatrix} \widetilde{e}_{1,1} & \cdots & e_{1,k} & \cdots & \widetilde{e}_{1,K} \\ \vdots & & \vdots & & \vdots \\ \widetilde{e}_{B,1} & \cdots & e_{B,k} & \cdots & \widetilde{e}_{B,K} \end{bmatrix}}_{E^{(k)}} \rightarrow \begin{matrix} \varpi_1^{(k)} \\ \vdots \\ \varpi_B^{(k)} \end{matrix}$$

Blue column kept, rest redrawn: keeps interactions. $B(K+1)$ runs total.

Step 3c — the measure of importance

$$I_k = \sqrt{\langle \varpi \, \varpi^{(k)} \rangle - \langle \varpi \rangle^2} \qquad (k = T_{e,0} \text{ here})$$

Measures of Importance — Homma & Saltelli, *J. Nucl. Sci. Technol.* **32** (1995)

- paired scores move together $\Rightarrow$ large I_k $\Rightarrow$ input k matters,
- unrelated scores $\Rightarrow$ $I_k \approx 0$ $\Rightarrow$ input k is negligible,
- a reliable I_k needs many simulations $\Rightarrow$ large B.

Results — two discharges analysed

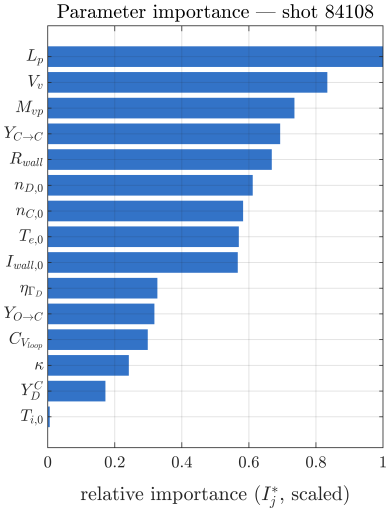
Shot 84108 — burn-through

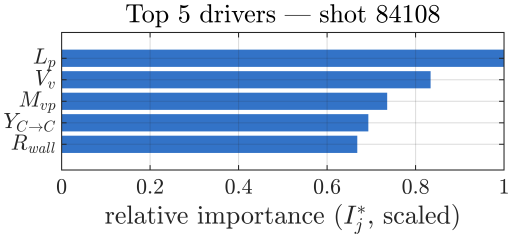
- no runaways
- 23 inputs
- 8 outputs

Shot 87090 — runaways

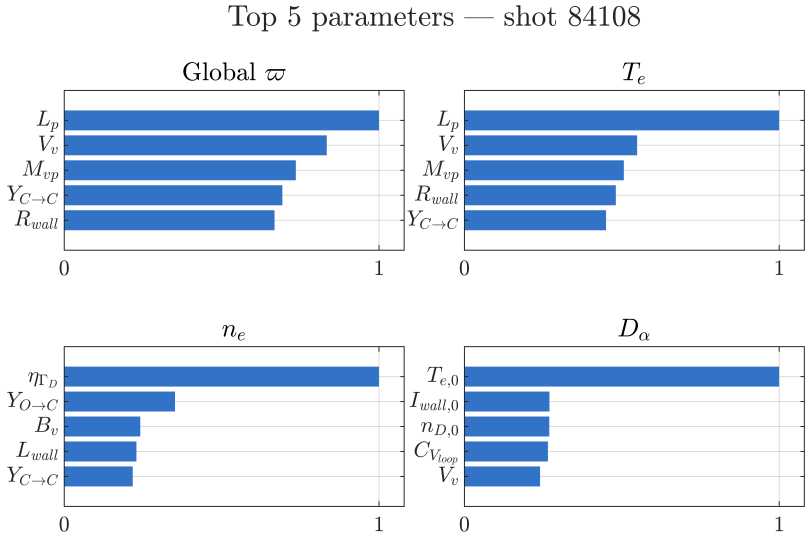
- no gas fuelling $\rightarrow$ runaways
- 5 inputs
- extra output: n_{RE}

Burn-through shot — only a handful of inputs matter

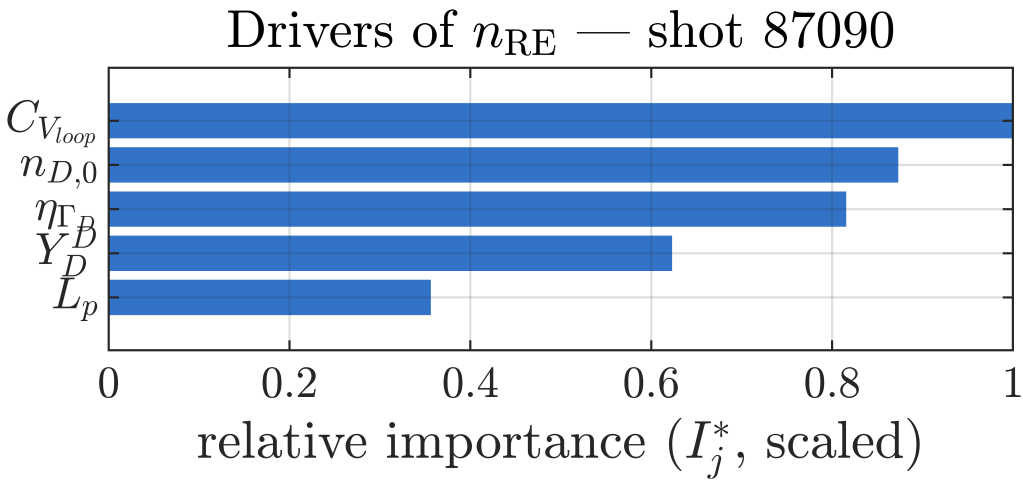




Burn-through shot — comparison



Runaway — what drives n_{RE}



Results — takeaways

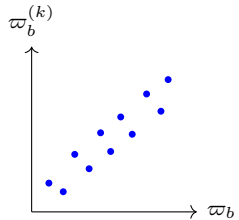
- **Burn-through discharge:**
  - High dimensionality $\rightarrow$ lower dimensionality
  - Stiff: $L_p,\ V_v,\ M_{vp}$
  - Sloppy: $T_{i,0}$
- **Runaway discharge:**
  - The 5-input study should be extended.
  - V_{loop} is the leading driver, followed by $n_{D,0}$

Closing

Questions?

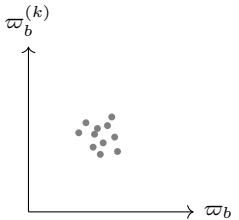
Appendix — reading the pairs: correlated or not

Each point is a $(\text{run}, \text{companion})$ pair, $\varpi_b^{(k)}$ vs ϖ_b (example: $T_{e,0}$ frozen):



correlated cloud

$T_{e,0}$ **matters**



uncorrelated cloud

$T_{e,0}$ **irrelevant**

$I_k = \sqrt{\langle \varpi\, \varpi^{(k)} \rangle - \langle \varpi \rangle^2}$ large for a tilted (correlated) cloud, near zero for a round one.

Appendix — why B must be large I

The example $B = 3$ only illustrates the matrix construction; it is far too small for a reliable estimate. Following Saltelli et al. (2000, pp. 171–173):
Let $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_K)$ be the uncertain inputs and $\varpi(\boldsymbol{\theta})$ the scalar discrepancy score of one run. For input k,

$$I_k^2 = \mathrm{Var}(\varpi) - \mathbb{E}[\mathrm{Var}(\varpi \mid \theta_k)] = U_k - (\mathbb{E}(\varpi))^2,$$

$$U_k = \int (\mathbb{E}(\varpi \mid \theta_k))^2 \, p_k(\theta_k) \, \mathrm{d}\theta_k,$$

where p_k is the marginal distribution of θ_k.
For $b = 1, \ldots, B$, let $\boldsymbol{\theta}^{(b)}$ be the b-th base sample and $\boldsymbol{\theta}^{(b,k)}$ the resampled configuration (θ_k fixed, the rest redrawn independently), with

$$\varpi_b = \varpi\left(\boldsymbol{\theta}^{(b)}\right), \qquad \varpi_b^{(k)} = \varpi\left(\boldsymbol{\theta}^{(b,k)}\right).$$

Appendix — why B must be large II

U_k has no closed form and full Monte Carlo is impractical, so Saltelli et al. (1993) estimate

$$\widehat{U}_k = \frac{1}{B}\sum_{b=1}^{B} \varpi_b \, \varpi_b^{(k)}, \qquad \mathbb{E}[\varpi] = \langle \varpi \rangle = \frac{1}{B}\sum_{b=1}^{B} \varpi_b,$$

$$\widehat{I}_k^2 = \widehat{U}_k - \langle \varpi \rangle^2 = \frac{1}{B}\sum_{b=1}^{B} \varpi_b \, \varpi_b^{(k)} - \left(\frac{1}{B}\sum_{b=1}^{B} \varpi_b \right)^2 .$$

Appendix — why B must be large III

Convergence (law of large numbers). If the base samples are i.i.d. and the resampled configurations are drawn independently, then the products $\varpi_b\,\varpi_b^{(k)}$ are i.i.d., so

$$\frac{1}{B}\sum_{b=1}^{B}\varpi_b\,\varpi_b^{(k)} \xrightarrow[B\to\infty]{} \mathbb{E}\Big[\varpi\,\varpi^{(k)}\Big] = U_k,$$

$$\frac{1}{B}\sum_{b=1}^{B}\varpi_b \xrightarrow[B\to\infty]{} \mathbb{E}(\varpi).$$

Hence

$$\widehat{I}_k^2 \xrightarrow[B\to\infty]{} U_k - \big(\mathbb{E}(\varpi)\big)^2 = I_k^2.$$

Appendix — why B must be large IV

Increasing B improves the statistical stability of the estimator. In practice, B is increased until the $\widehat{I}_k$ no longer change significantly when additional samples are added.

References: Saltelli, Chan & Scott, *Sensitivity Analysis* (2000); Saltelli, Andres & Homma, *Comput. Phys. Commun.* (1993).