Simulation of Anomalous Radiation of Runaway Electrons Induced by External Injection of Helicon Waves

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Thanks to:
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Hari Choudhury
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Contents

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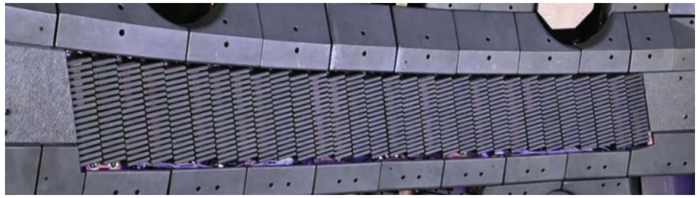
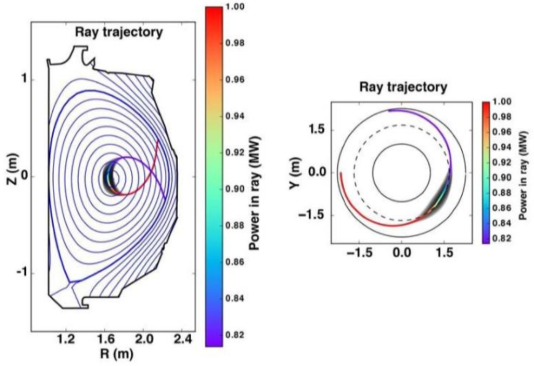
Physics Motivation : whistler waves limit REs’ growth

- During tokamak startup and disruption, a large number of runaway electrons are generated and flows toward the first wall, causing severe damage to the plasma-facing core components.
- Runaway electrons can be resonant scattered by **whistler waves**, helps limit the REs’ energy and density growth.

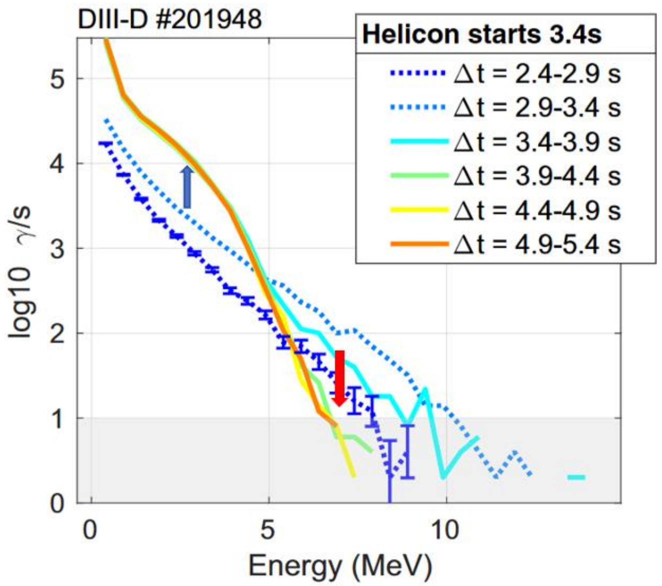
Example:DIII-D helicon wave inject experiment



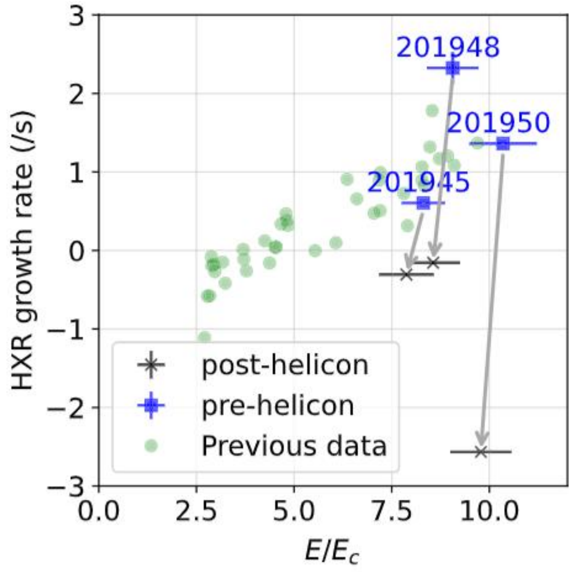


(b) (c)

DIII-D 476MHz helicon wave attenna and Waves propagation
R.I. Pinsker et al 2024 Nucl. Fusion 64 126058



high energy runaway electron numbers are reduced.



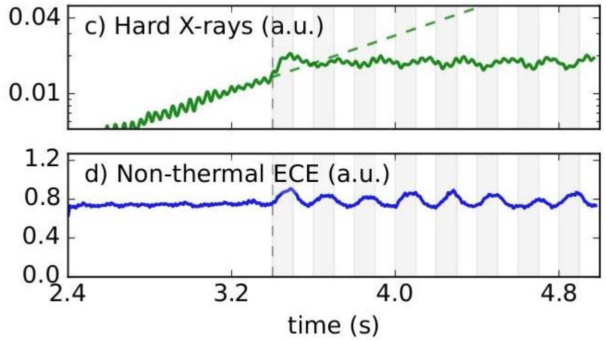
In DIII-D, after helicon wave injection, HXR growth rate turns negative.
Hari Choudhury. PHYSICAL REVIEW LETTERS 136, 025101 (2026)

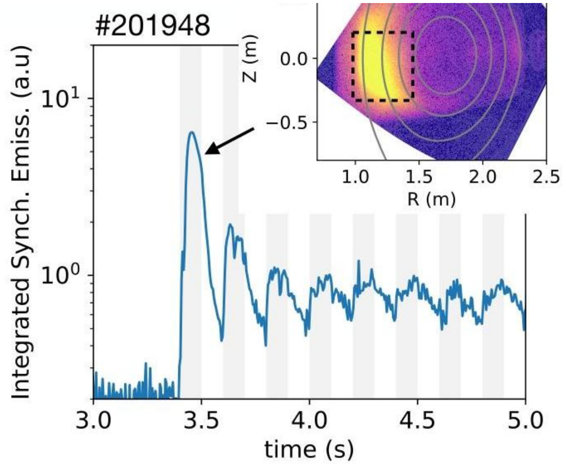
Experiment results summary

Helicon wave injection in DIII-D Quiescent Runaway Electron (QRE, E/Ec≈ 10) experiments use 476MHz helicon wave inject to control REs.

Helicon wave inject

- Hard X-ray (HXR) : stop rising. → avalanche supressed.
- Non thermal ECE: increases by about 10% -20% during wave pulse. → low-energy runaway electrons generation
- Synchrotron radiation: pulses up for the first time, then gradually converging → high energy runaway electrons are scatterd





The runaway electron population has reached a balance between generation and loss after helicon wave periodically injection.

Possible reasons:

1. Avalanche is suppressed due to the increase critical Electric field E_c after wave scattered.
2. Scattered runaway electrons have higher lost rate.

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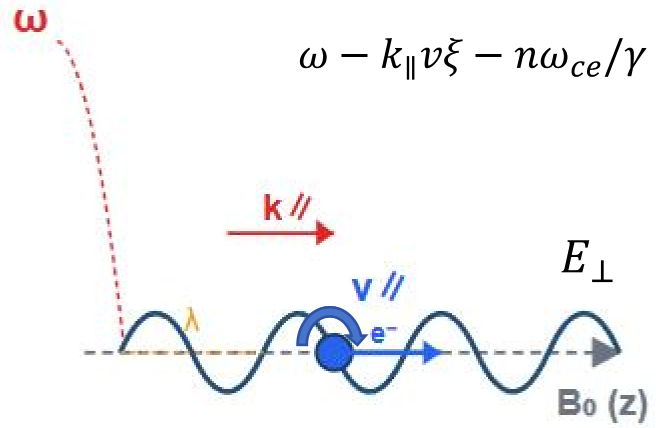
Wave-Particle Resonance increase pitch-angle

What has been down: Helicon waves selectively scatter high-energy runaway electrons via normal Doppler resonance, increasing their pitch angle and synchrotron radiation losses, helps Runaway Eletron vortex formation.

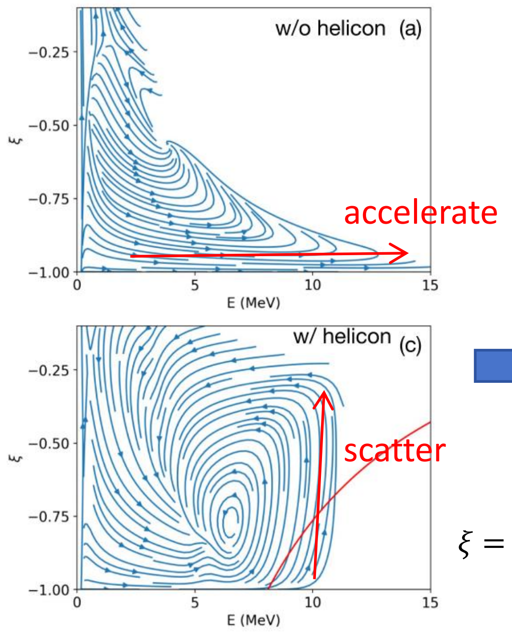
Doppler shift cycrotron Resonance condition

$$\omega - k_{\parallel} v \xi - n\omega_{ce}/\gamma = 0$$



Cyclotron resonance increase $v_\perp$

RE vortex means scattered REs flows back to low energy, calculated by LAPS-RFP code.



$\xi = \cos\theta$

Lack of radiation simulation

[1]Zehua Guo et al. Phys. Plasmas 25, 032504 (2018)
[2]H. Choudhury. PHYSICAL REVIEW LETTERS 136, 025101 (2026)

Simulation Framework : DREAM

We choose DREAM to simulate the whole process, including runaway electron generation and wave-particle scattering.

DREAM models electron dynamics with a **bounce-averaged drift-kinetic** equation of the form

$$\frac{\partial f}{\partial t} = \sum_{m,n} \frac{1}{\mathcal{V}'} \frac{\partial}{\partial z^m} \left[\mathcal{V}' \left(-\{A^m\} f + \{D^{mn}\} \frac{\partial f}{\partial z^n} \right) \right] + \{S\}$$

The bounce average bracket $\{X\}$ integrates X in one complete bounce/passing period.

Terms in the equation :

$$A = \boldsymbol{A}_E + \boldsymbol{A}_C + \boldsymbol{A}_B + \boldsymbol{A}_S + \boldsymbol{A}_T \quad \boldsymbol{D} = \boldsymbol{D}_C + \mathbf{D}_{\boldsymbol{T}} + \mathbf{D}_{\mathbf{QL}}$$

Terms exist before: Electric field $\boldsymbol{A}_E$, collisional friction $\boldsymbol{A}_C$, bremsstrahlung friction $\boldsymbol{A}_B$, synchrotron friction $\boldsymbol{A}_S$, radial transport $\boldsymbol{A}_T$, collisional diffution $\boldsymbol{D}_C$, radial transport diffution $\mathbf{D}_{\boldsymbol{T}}$.

We add new terms: Quasilinear Diffusion term $\mathbf{D}_{\mathbf{QL}}$, representing the resonance diffusion of whistler waves on electrons.

Mathias Hoppe et al. Computer Physics Communications,Volume 268,2021,108098.

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Quasi-Linear Diffusion modelling of Wave-Particle Resonance

In p-xi axis, Diffusion Term can be generally modelled by

$$\frac{\partial f}{\partial t}=\frac{1}{p^2}\frac{\partial}{\partial p}p^2\left[D_{pp}\frac{\partial f}{\partial p}+\frac{\sqrt{1-\xi^2}}{p}D_{p\xi}\frac{\partial f}{\partial \xi}\right]\quad+\frac{1}{p}\frac{\partial}{\partial \xi}\left[\sqrt{1-\xi^2}D_{\xi p}\frac{\partial f}{\partial p}+\frac{1-\xi^2}{p}D_{\xi\xi}\frac{\partial f}{\partial \xi}\right]$$

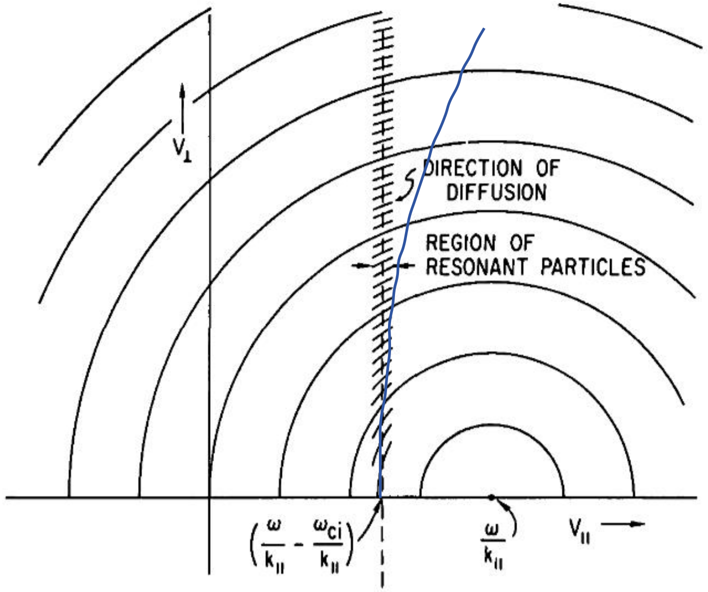
Quasi-Linear Diffusion components are non-zero near **resonance condition**:

$$D_{pp}=\frac{\pi e^2}{\varepsilon_0 m_e^2}(1-\xi^2)\sum_n\int d^3k\,\frac{|\mathbf{E}_{k,n}|^2}{\omega_k}\,|\Psi_{n,k}|^2\,\delta\left(\omega_k-k_\parallel v_\parallel-\frac{n\omega_{ce}}{\gamma}\right),$$

$$D_{p\xi}=-\frac{\pi e^2}{\varepsilon_0 m_e^2}\sqrt{1-\xi^2}\sum_n\int d^3k\,\frac{|\mathbf{E}_{k,n}|^2}{\omega_k}\,|\Psi_{n,k}|^2\,\delta\left(\omega_k-k_\parallel v_\parallel-\frac{n\omega_{ce}}{\gamma}\right)\left(\xi-\frac{k_\parallel v}{\omega_k}\right),$$

$$D_{\xi\xi}=\frac{\pi e^2}{\varepsilon_0 m_e^2}\sum_n\int d^3k\,\frac{|\mathbf{E}_{k,n}|^2}{\omega_k}\,|\Psi_{n,k}|^2\,\delta\left(\omega_k-k_\parallel v_\parallel-\frac{n\omega_{ce}}{\gamma}\right)\left(\xi-\frac{k_\parallel v}{\omega_k}\right)^2.$$

relativistic resoncant area



Direction for quasi-linear diffusion is tangential to the concentric circles. Stix(1975)

electric field spectral density:

$$\langle|E_n(\mathbf{r},t)|^2\rangle=\int d^3k\,|E_{\mathbf{k},n}|^2$$

wave-particle coupling coefficient:

$$|\Psi_{n,k}|^2=\left|\frac{n}{z}J_n(z)-\frac{k_\parallel}{k}J_n'(z)+\frac{p_\parallel}{p_\perp}\frac{k_\parallel k_\perp c^2}{\omega_{pe}^2}J_n(z)\right|^2,\qquad z=k_\perp\frac{p\sqrt{1-\xi^2}}{m_e\omega_{ce}}$$

Zehua Guo et al. Phys. Plasmas 25, 032504 (2018)

Numerical process of Quasi-Linear Diffusion Matrix

To calculate Quasi-Linear Diffusion in DREAM, we need to process delta function

$$\int dk\, \delta\Big(\omega(k,\theta_k) - k_\parallel v_\parallel - \frac{n\omega_{ce}}{\gamma}\Big)\, \mathcal{H}(k,\theta_k) = \frac{\mathcal{H}(k,\theta_k)}{\left|\dfrac{\partial\omega}{\partial k} - v_\parallel \cos\theta_k\right|}_{k=k_{\mathrm{res}}}$$

$$D_{pp} = \frac{\pi e^2}{\varepsilon_0 m_e^2}(1-\xi^2)\sum_n \int d^3k\, \frac{|\mathbf{E}_{k,n}|^2}{\omega_k}|\Psi_{n,k}|^2\, \delta\left(\omega_k - k_\parallel v_\parallel - \frac{n\omega_{ce}}{\gamma}\right), \Rightarrow D_{pp}(p,\xi) = (1-\xi^2)\frac{\pi e^2}{\varepsilon_0 m_e^2}\sum_n \int_0^\pi \sin\theta_k\, d\theta_k \left[\frac{k^2\, |\mathbf{E}_{k,n}|^2\, |\Psi_{n,k}|^2}{\omega_k \left|\dfrac{\partial\omega}{\partial k} - v_\parallel\cos\theta_k\right|}\right]_{k=k_{\mathrm{res}}(p,\xi,\theta_k,n)}$$

In DREAM FVM framework, $\mathbf{D_{pp}}$, $\mathbf{D_{p\xi}}$, $\mathbf{D_{\xi\xi}}$ need to be calculated and assembled into the flux grid.

Summation kernel

$$\mathcal{K}_{i,j}^{(n)}(k,\theta_k) \equiv \frac{2\pi^2 e^2}{\varepsilon_0 m_e^2}\, k^2 \sin\theta_k\, \frac{|\mathbf{E}_{k,n}|^2}{\omega_k}\, |\Psi_{n,k}|^2\, \frac{w_i}{|\partial\omega/\partial k - v_\parallel\cos\theta_k|}\,\Delta k\,\Delta\theta_k,$$

Use w_i to find (p,ξ) where $\mathbf{D_{QL}}$ is non-zero)

$$w_i = \max\left(0,\ 1 - \frac{|p_{\mathrm{res}} - p_i|}{\Delta p}\right)$$

$$D_{pp}(p_i,\xi_j) = (1-\xi_j^2)\sum_n\sum_{k,\theta}\mathcal{K}_{i,j}^{(n)}(k,\theta_k),$$

$$D_{p\xi}(p_i,\xi_j) = -\sqrt{1-\xi_j^2}\sum_n\sum_{k,\theta}\mathcal{K}_{i,j}^{(n)}(k,\theta_k)\left(\xi_j - \frac{k_\parallel v_{ij}}{\omega_k}\right),$$

$$D_{\xi\xi}(p_i,\xi_j) = \sum_n\sum_{k,\theta}\mathcal{K}_{i,j}^{(n)}(k,\theta_k)\left(\xi_j - \frac{k_\parallel v_{ij}}{\omega_k}\right)^2.$$

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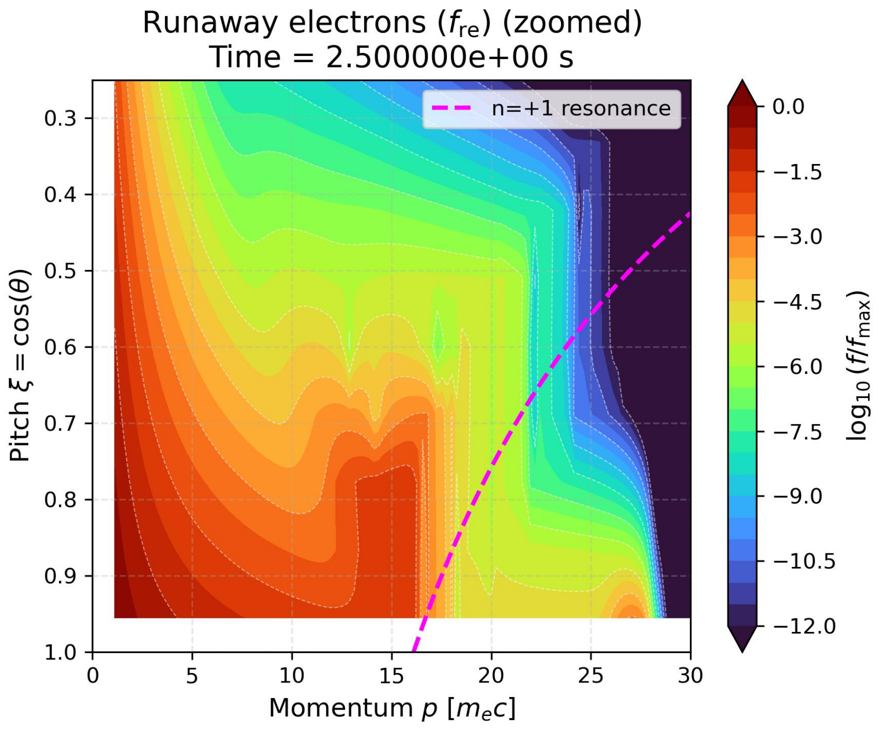
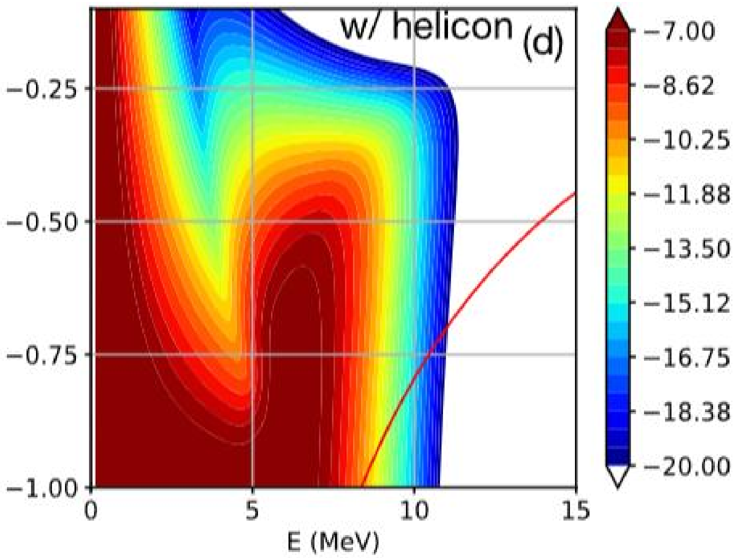
Benchmark with LAPS-RFP code

LAPS-RFP code is the first code calculate how inject whistler wave form RE vortex cutting off the high-energy part of the runaway electrons.Our DREAM simulation result can also see this RE vortex, almost same position (near resonance line) and size with LAPS-RFP result.



DREAM (eps=0.01)



LAPS-RFP

Red line: resonant condition

[1]Zehua Guo et al. Phys. Plasmas 25, 032504 (2018)
[2]H. Choudhury. PHYSICAL REVIEW LETTERS 136, 025101 (2026)

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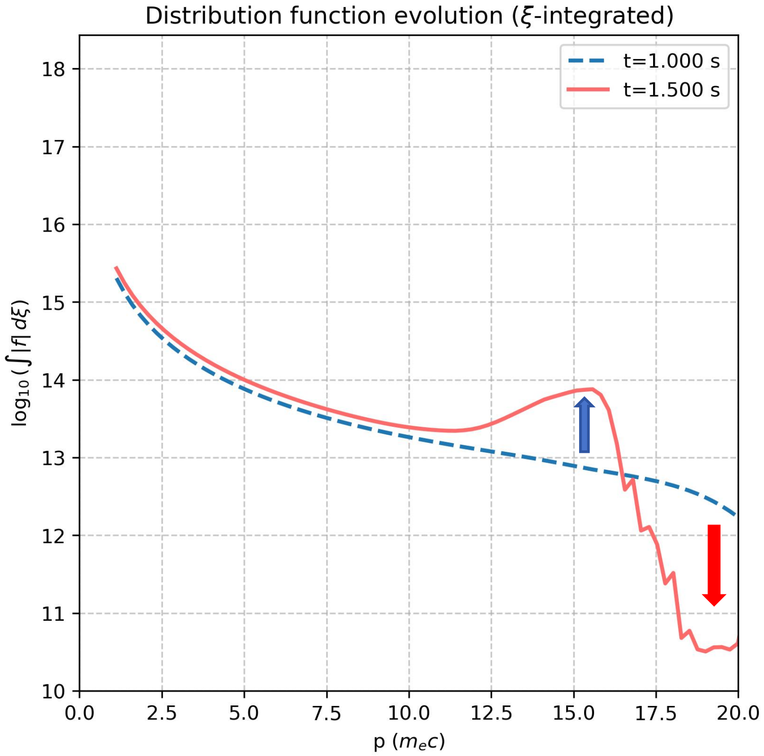
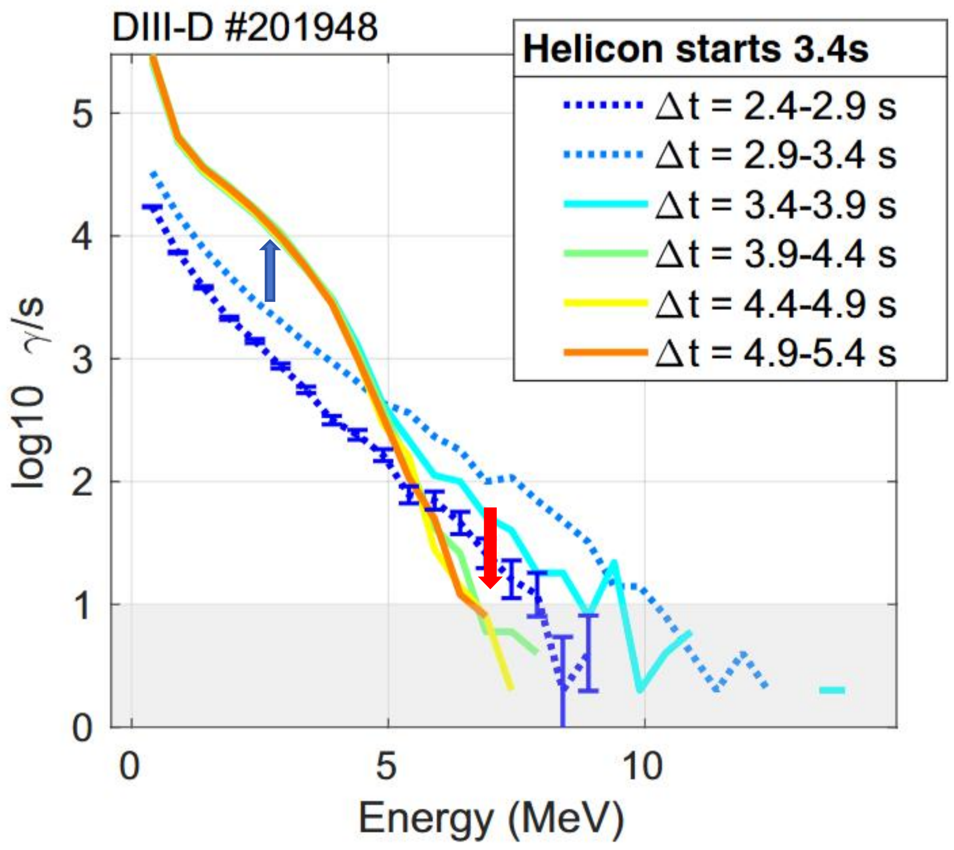
RE Distribution modified by helicon wave injection

Simulation results shows an decrease of runaway electrons above the resonant energy, consistent with experiment HXR diagnostics.

Simulation Results





The spectra of HXRs emitted by the REs for different time windows.

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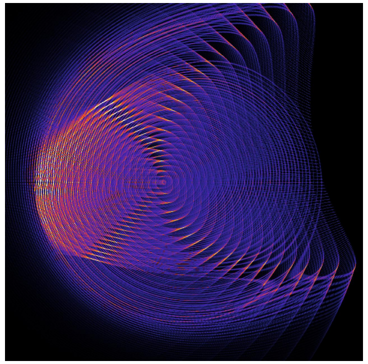
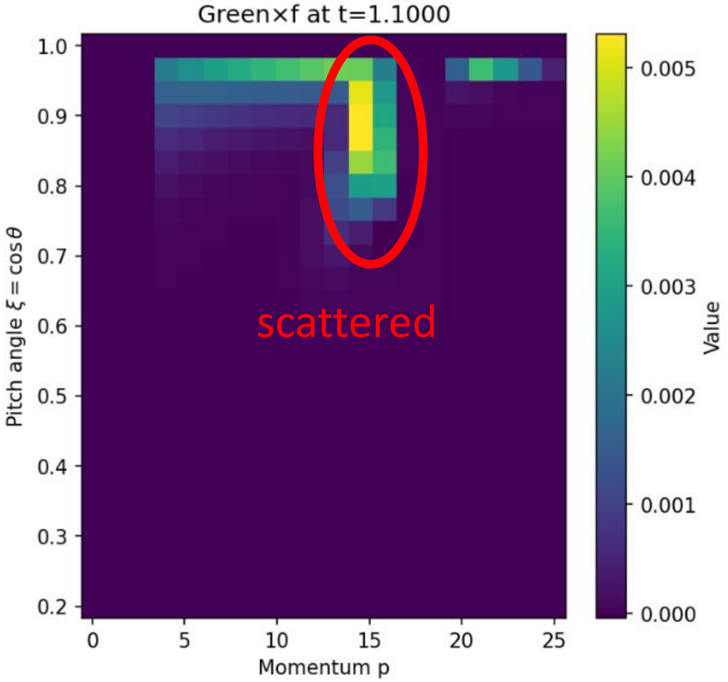
Synchrotron Radiation Enhancement due to scattered REs

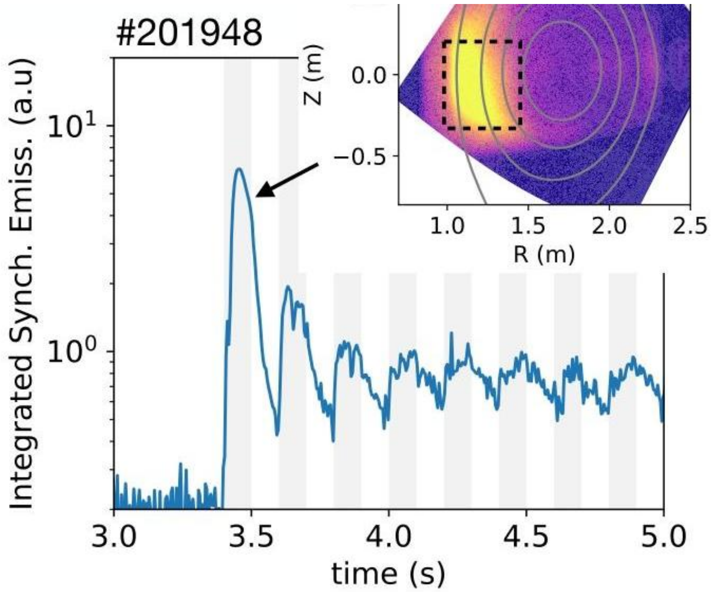
Synchrotron Radiation is contributed by high energy REs.
Helicon wave scattered REs, increase synchrotron sadiation.

Simulation Results (SOFT)

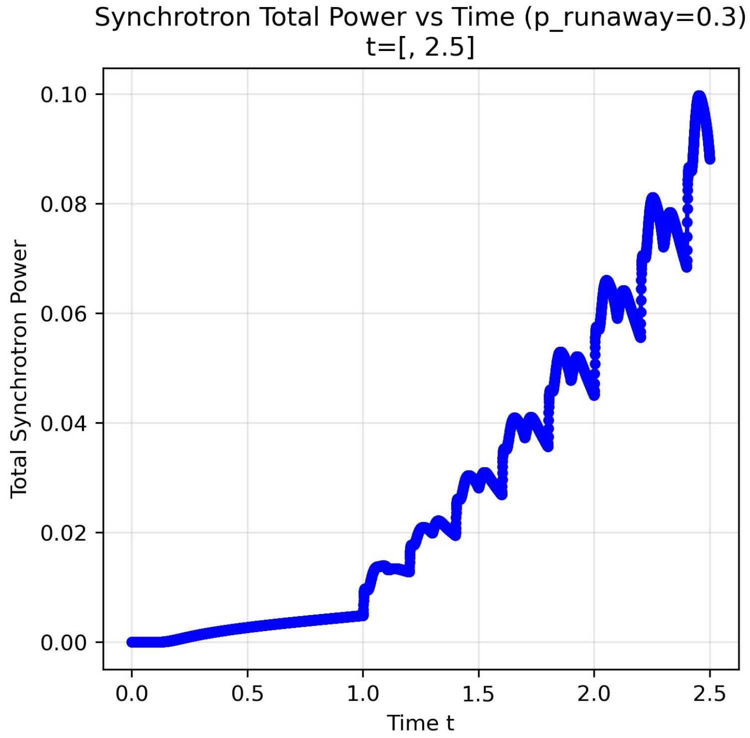




Synchrotron weight is dominated by the scattered electrons after helicon wave injection



Synchrotron radiation in experiment pulses up for the first time, then gradually converging in experiment.



Simulation results (SOFT) shows synchrotron power rises sharply after 1st wave burst, then continues to grow due to ongoing RE generation.

SOFT program: https://github.com/hoppe93/SOFT2

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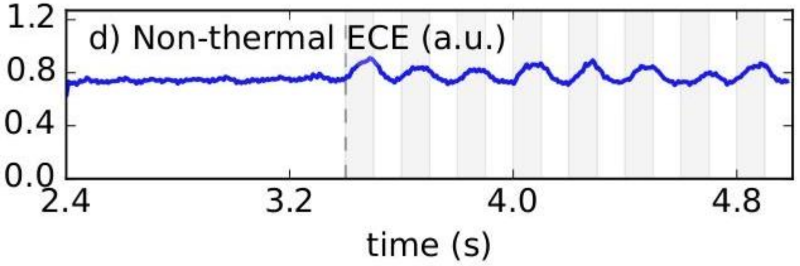
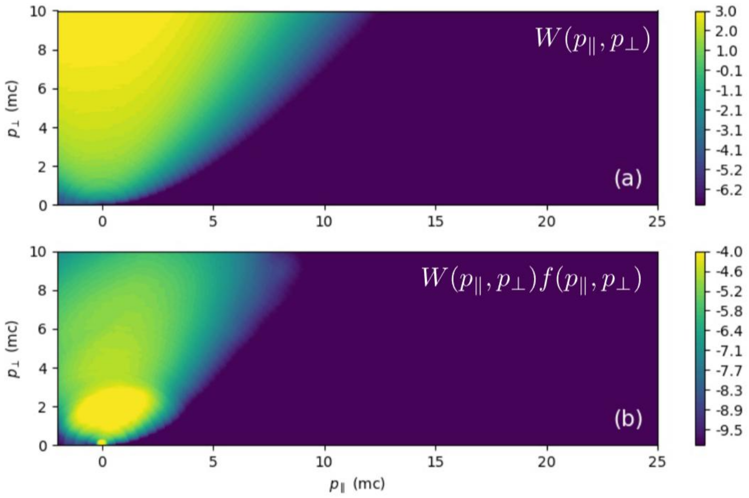
ECE: enhanced by low energy RE

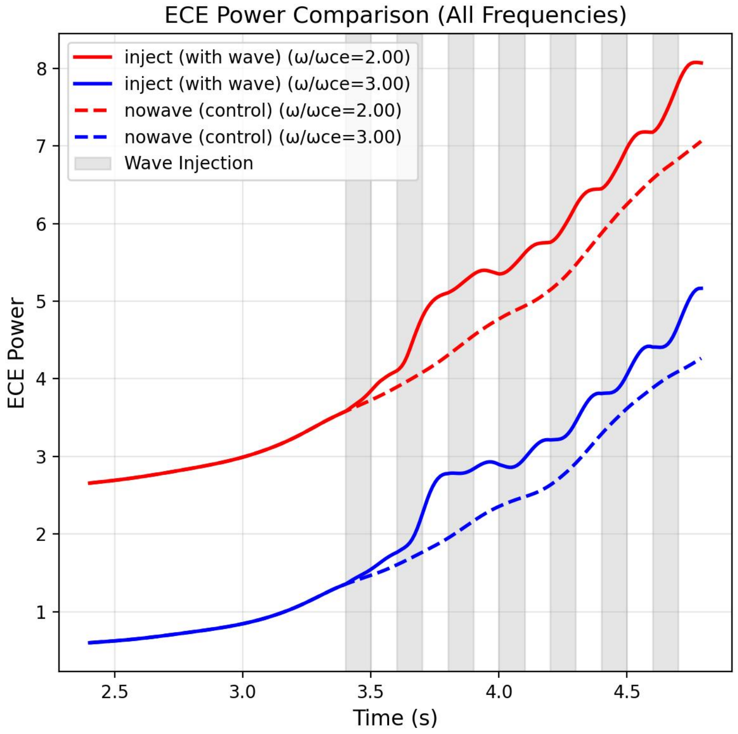
ECE radiation power is contributed by thermal and non-thermal electrons.
Runaway electrons with low $p_{\parallel}$ and a finite $p_{\perp}$ contribute growth of ECE signals.



In the experiment, the non thermal ECE increased by about 10% -20% during wave pulse period



(a) Phase space region of $3\omega_{ce}$ ECE signal contribution.



(b) ECE grow everytime when wave inject.

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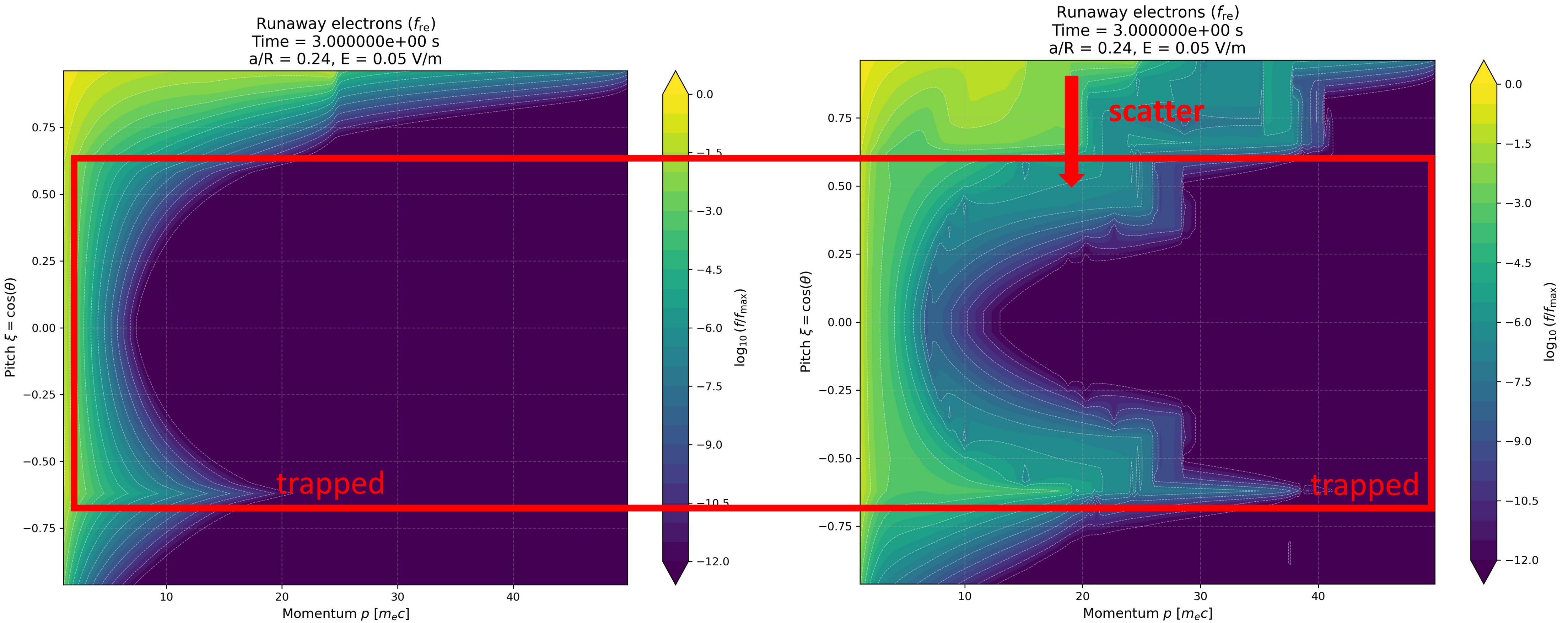
Scattering by helicon wave can increase trapping fraction

Using DREAM simulation with finite a/R, we find helicon wave can scatter REs into trapped region

- Majority of REs are still passing



without wave inject

with wave inject

Electron distribution function in momentum space

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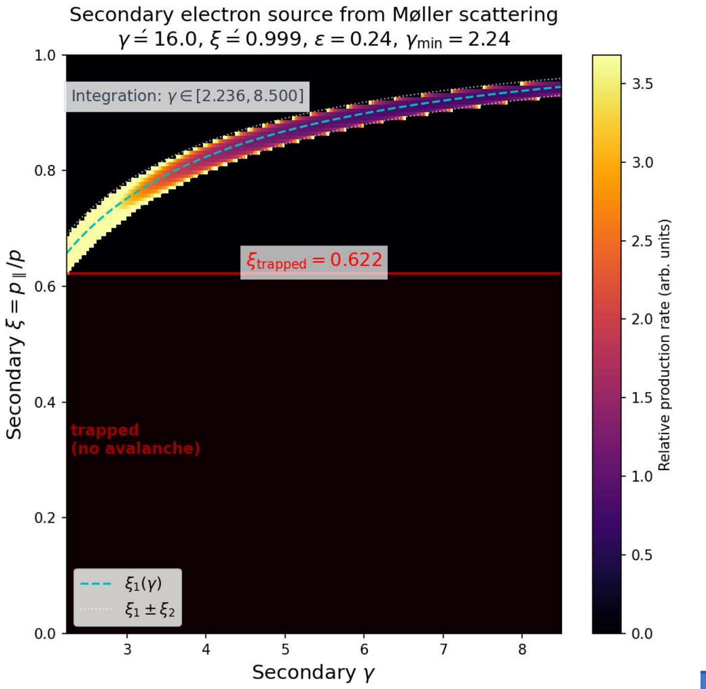
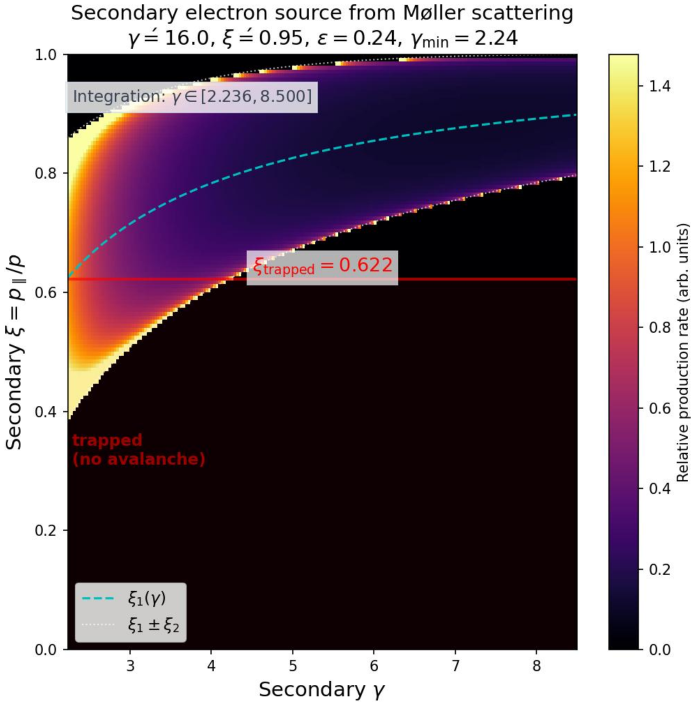
Decrease of avalanche rate due to finite pitch angle of primary electron

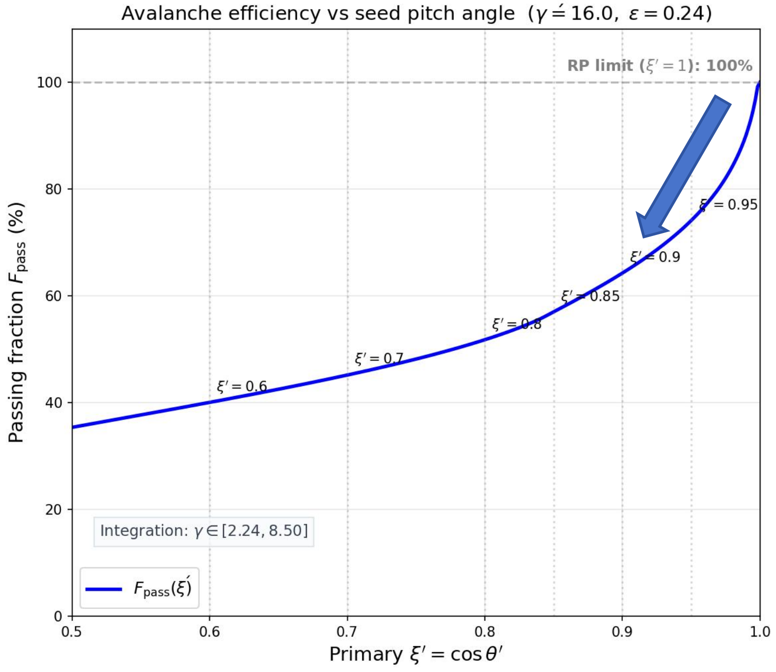
- Rosenbluth-Putvinski model works while primary electron has $\xi \to 1$
- In more accurate Møller scattering model, primary electron's pitch angle affects secondary generation

$$\mathrm{density}(\gamma,\xi) \propto \frac{d\sigma_M(\gamma',\gamma)}{d\gamma} \times \Pi(\gamma',\xi',\gamma;\xi) \times \frac{\gamma^2}{\gamma^2-1}. \qquad \Pi(\xi;p_1,\xi_1,p) = \frac{1}{\pi\sqrt{\xi_2^2-(\xi-\xi_{1c})^2}}, \quad |\xi-\xi_{1c}|<\xi_2 \qquad \xi_2 = \sqrt{\frac{2(\gamma_e-\gamma)}{(\gamma_e-1)(\gamma+1)}(1-\xi_e^2)}$$

When primary electron pitch angle scatters, ξ' decrease, large number of secondary electrons will be trapped.







Summary

- We developed DREAM Quasilinear Diffusion term to simulate the process of helicon wave scattering high-energy runaway electrons to large pitch angles. Quasilinear diffusion causes synchrotron radiation and ECE rise, consistent to DIII-D experiment result.
- Synchrotron force will deaccelerate scattered runaway electrons. ECE's increase may be caused by those already decelerated scattered electrons.
- Although waves scatter some REs into trapped region, major REs are still passing, so trapping effect alone cannot provide an explanation of avalanche suppression.
- There still remains quesions of how helicon wave supress RE avalanche rate, and how RE lost after wave scattering.

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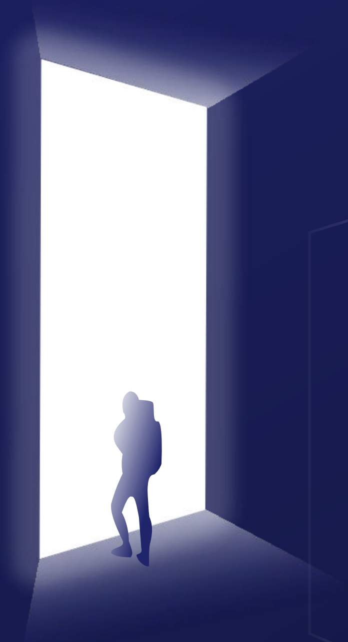
THANK YOU



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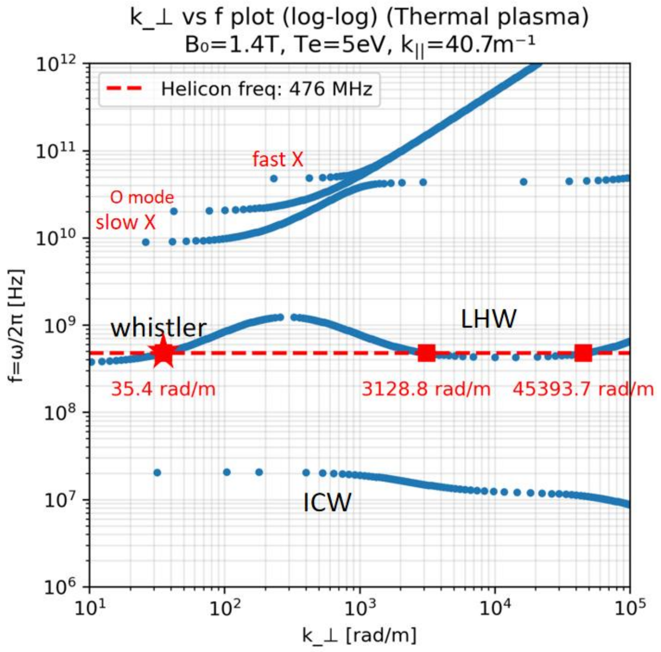
Appendix



476MHz Helicon wave is at Whistle branch, electromagnetic branch. Dispersion code BON: https://github.com/hsxie/bon

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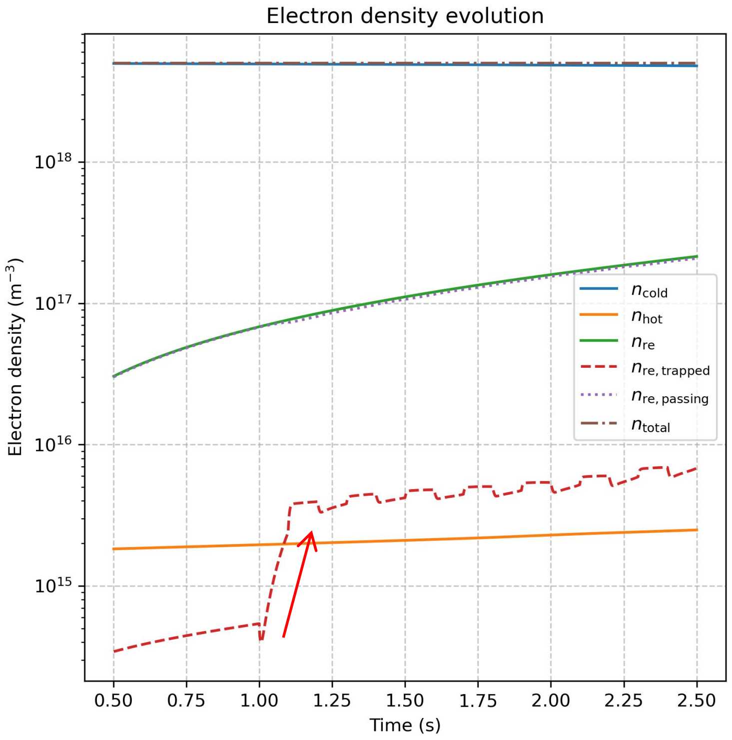
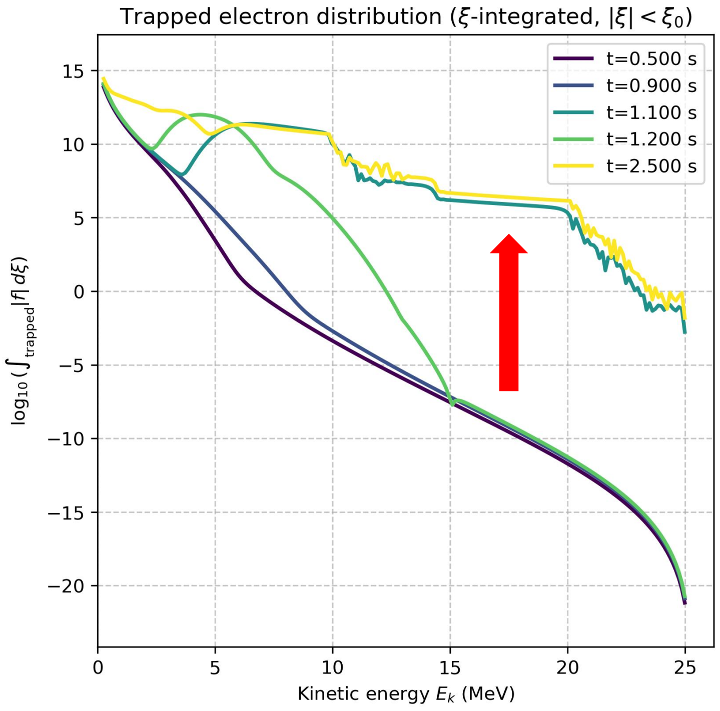
Wave-Induced Scattering Enhances Trapped Electron Population

The total runaway density (left) remains almost unchanged, but the trapped fraction (right) increases due to pitch-angle scattering of high-energy REs.





Whistler wave scattered REs increase High Energy trapped REs. (eps=a/R=0.24)

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