Magnetic field threshold for runaway generation in tokamak disruptions

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Experimental observations show that there is a magnetic field threshold for runaway electron generation in tokamak disruptions. In this work, two possible reasons for this threshold are studied. The first possible explanation for these observations is that the runaway beam excites whistler waves that scatter the electrons in velocity space prevents the beam from growing. The growth rates of the most unstable whistler waves are inversely proportional to the magnetic field strength. Taking into account the collisional and convective damping of the waves it is possible to derive a magnetic field threshold below which no runaways are expected. The second possible explanation is the magnetic field dependence of the criterion for substantial runaway production obtained by calculating how many runaway electrons can be produced before the induced toroidal electric field diffuses out of the plasma. It is shown, that even in rapidly cooling plasmas, where hot-tail generation is expected to give rise to substantial runaway population, the whistler waves can stop the runaway formation below a certain magnetic field unless the postdisruption temperature is very low. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3072980]

I. INTRODUCTION

Due to a sudden cooling of the plasma in tokamak disruptions a beam of relativistic runaway electrons is sometimes generated, which can cause damage on plasma facing components due to highly localized energy deposition. This problem becomes more serious in larger tokamaks with higher plasma currents and understanding of the processes that may limit or eliminate runaway electron generation is very important for future tokamaks, such as, ITER.[1] In present tokamak experiments it is observed that the number of runaway electrons generated depends on the magnetic field strength. Several large tokamaks have reported that no runaway generation occurs unless the magnetic field B exceeds 2 T.[2,3] Above this threshold, the runaway generation shows a nonlinear dependence on B, and a doubling of B results in an increase of the photo-neutron production by two orders of magnitude.[4]

In this work, two possible reasons for the observed magnetic field threshold are studied. The first explanation is associated with the whistler wave instability (WWI) which can be excited by runaway electrons. The growth rates of the most unstable whistler waves are inversely proportional to the magnetic field strength, and the WWI causes a rapid pitch-angle scattering of the runaways which may stop the runaway beam formation.[5,6] The second explanation is related to the magnetic field dependence of the criterion for runaway avalanche (CRASH), which can be derived from the coupled dynamics of plasma current and runaway generation. This theoretical criterion for substantial runaway generation was derived in Refs. 7 and 8, but without the effect of the hot-tail generation of runaways.[9–13] The aim of the present work is to compare the magnetic field dependence of the threshold for WWI with the one of CRASH and analyze the relevance of the two different mechanisms for the Joint European Torus (JET),[14] and ITER-like disruption scenarios. We generalize the threshold criterion for the WWI derived in Ref. 5 by taking into account the localization of the runaway beam that gives rise to convective damping of the waves. We also generalize CRASH derived in Ref. 8 by including the hot-tail generation of runaways. The main result of the paper is that even in rapidly cooling plasmas, where hot-tail generation is expected to give rise to substantial runaway population, the whistler waves can stop the runaway formation below a certain magnetic field (of the order of 2 T in the JET) unless the postdisruption temperature is very low (below 10 eV).

The structure of the paper is the following: In Sec. II the stability threshold of the whistler waves driven by relativistic runaway electrons, as well as the most unstable wave frequency and wave number are given, based on the analysis in Refs. 5 and 6 but including the effect of convective damping. In Sec. III the criterion for substantial runaway electron generation derived in Refs. 7 and 8 is generalized to include hot-tail generation and its dependence on the cooling time is discussed. In Sec. IV the linear threshold for the WWI is compared with the criterion for substantial runaway generation and their relevance is discussed for JET and ITER-like scenarios. Finally, the conclusions are summarized in Sec. V.

II. WWI

The runaway electron beam has a strongly anisotropic velocity distribution. When the degree of anisotropy exceeds a critical level, unstable whistler waves, with frequencies well below the nonrelativistic electron cyclotron frequency ω_{ce} but above the ion cyclotron frequency ω_{ci} are excited.

Assuming $k_\perp^2 v_{Te}^2 \ll \omega^2$, where v_{Te} is the electron thermal velocity, the dispersion relation of these waves can be simplified to[6]

$$k^2 v_A^2\left(1+\frac{k_\|^2 v_A^2}{\omega_{ci}^2}+\frac{k_\|^2}{k^2}\right)-\omega^2\left[1+\frac{(k^2+k_\|^2)v_A^2}{\omega_{ci}\omega_{ce}}\right] = \omega^2\frac{\omega_{ci}^2}{\omega_{pi}^2}\left[\left(1+\frac{k^2 v_A^2}{\omega_{ci}^2}\right)\chi_{11}^r+\left(1+\frac{k_\|^2 v_A^2}{\omega_{ci}^2}\right)\chi_{22}^r + 2i\frac{\omega}{\omega_{ci}}\chi_{12}^r\right], \quad (1)$$

where ω is the wave frequency, $k_\|$ and $k_\perp$ are the parallel and perpendicular components of the wave number, $v_A=c\omega_{ci}/\omega_{pi}$ is the Alfvén velocity, ω_{pi} is the ion plasma frequency, c is the speed of light, χ_{ij}^r denotes the runaway contribution to the susceptibility tensor. Without runaways, for $k_\|^2 c^2/\omega_{pi}^2 \gg 1$ and $(k^2+k_\|^2)v_A^2 \ll \omega_{ci}\omega_{ce}$, the dispersion relation can be further simplified to obtain the usual relation for the whistler wave $\omega = k k_\| v_A^2/\omega_{ci}$.

Numerical simulations in Ref. 5 showed that the most important interaction occurs at the anomalous Doppler resonance $\omega - k_\| v_\| = -\omega_{ce}/\gamma$, where $v_\|$ is the particle velocity parallel to the magnetic field and γ is the relativistic factor. The linear growth rates of these waves are such that they are stable for high magnetic field (so the runaway beam can form) but unstable for low magnetic field. In Ref. 5 it was shown that the linear growth rate of a small perturbation $\omega=\omega_0+\delta\omega$ is $\gamma_i = \mathrm{Im}\,\delta\omega = (k-k_\|)^2 v_A^2 \omega_0 \,\mathrm{Im}\,\chi^r(t)/2\omega_{pi}^2$, with

$$\chi^r(t) = \frac{\omega_{pr}^2\omega_{ce}^2}{k_\perp^2 \omega c^2}\int d^3p \frac{J_1^2(z)}{\omega(\omega\gamma - k_\| p_\| c + \omega_{ce})} \times\left(\frac{\omega - k_\| p_\| c/\gamma}{p_\perp}\frac{\partial f_r}{\partial p_\perp}+\frac{k_\| c}{\gamma}\frac{\partial f_r}{\partial p_\|}\right). \quad (2)$$

In this expression, $\omega_{pr}^2 = n_r e^2/m_{e0}\epsilon_0$, where n_r is the runaway density and J_n is the Bessel function of the first kind and order n with the argument $z=k_\perp p_\perp c/\omega_{ce}$. The quantity $f_r = f/n_r$ is the normalized secondary runaway distribution function[5]

$$f(p_\|, p_\perp, t) = \frac{\alpha n_r}{2\pi c_Z p_\|}\exp\left(-\frac{p_\|}{c_Z}-\frac{\alpha p_\perp^2}{2p_\|}\right), \quad (3)$$

where $\alpha=(\hat{E}-1)/(1+Z)$, p is the relativistic momentum, $\hat{E} = e|E|\tau/m_{e0}c$ is the normalized parallel electric field, $\tau = 4\pi\epsilon_0^2 m_{e0}^2 c^3/n_e e^4 \ln\Lambda$ is the collision time for relativistic electrons, m_{e0} is the electron rest mass, Z is the effective ion charge, and $c_Z = \sqrt{3(Z+5)/\pi}\ln\Lambda$. The distribution in Eq. (3) is valid if $\hat{E} \gg 1$ and secondary generation of runaways is dominant, as expected to be the case in large tokamak disruptions. The runaway density grows as $dn_r/dt = (\hat{E}-1)n_r/c_Z\tau$,[15] giving $n_r = n_{r0}\exp[(\hat{E}-1)t/(\tau c_Z)]$ if the electric field is assumed constant in time, and where n_{r0} denotes the seed produced by primary (Dreicer+hot-tail) generation.

Usually, in low-temperature plasmas, the dominant damping process is the electron-ion collisional damping, with $\gamma_d \simeq 1.5\tau_{ei}^{-1}$,[16] where $\tau_{ei} = 3\pi^{3/2} m_{e0}^2 v_{Te}^3 \epsilon_0^2/n_i Z^2 e^4 \ln\Lambda$ is the electron-ion collision time. If the runaway beam is localized with an effective beam radius L_r, then the wave-convection out from the region where the beam is localized gives a damping term $\gamma_v \equiv (\partial\omega/\partial k_\perp)/(4L_r) = v_A^2 k_\|/4\omega_{\mathrm{ci}} L_r$.[6] At low density, high temperature and strong magnetic field this term can be comparable to the collisional damping γ_d. The total linear growth rate of the WWI is then $\gamma_l = \gamma_i - \gamma_d - \gamma_v$.

The linear stability analysis in Ref. 5 has shown that the frequency, growth rate, and wave number of the most unstable wave are approximately $\omega_0 = \omega_{ce}/c_Z$, $\gamma_i = 1.3\times 10^{-9} n_r/B_T$, $k_0 = \omega_{pi}/2v_A = 3\times 10^4 n_{19} B_T\ \mathrm{m}^{-1}$, and $k_{\|0} = 2\omega_{ce}/(c c_Z) = 30 B_T\ \mathrm{m}^{-1}$, where B_T is the toroidal magnetic field in Tesla and $n_e = n_{19} 10^{19}\ \mathrm{m}^{-3}$ is the electron density. These values for the most unstable frequency and wave number have been calculated without taking into account the convective damping, the effect of which can be shown to reduce the values of k_0 and $k_{\|0}$ (still within the assumptions applied here) but the growth rate γ_i is quite insensitive on the exact magnitude of k_0 and $k_{\|0}$.[5]

From the linear instability threshold $\gamma_l > 0$ for the most unstable wave one can derive a threshold for the fraction of runaways required for destabilization

$$\frac{n_r}{n_e} > \frac{Z^2 B_T}{20 T_{\mathrm{eV}}^{3/2}} + \frac{B_T^3}{90 c_Z n_{19}^2 L_r}, \quad (4)$$

where T_{eV} is the postdisruption electron temperature in eV. Here, the first term on the right-hand side is due to the collisional damping and the second term is due to the convective damping of the WWI. The lower the magnetic field and higher the postdisruption temperature the less runaways are needed for the destabilization of WWI. The threshold presented in Eq. (4) is calculated using the wave numbers k_0 and $k_{\|0}$ given above. Numerical simulations show that the most unstable wave numbers are lower if convective damping is taken into account and slightly less runaway electrons are needed for destabilization.[6]

Figure 1 shows the stability threshold from Eq. (4) for different beam radii and fractions of runaways. The thickest line represents the case with a wide runaway beam so that convective damping is negligible. The thinner lines correspond to thinner runaway beams. Interestingly, the decreasing runaway beam radii lead to lower magnetic field thresholds for a given postdisruption temperature and these values for the magnetic field threshold become effectively independent of the temperature above a certain value. Note specifically in the upper figure, that the curve corresponding to the parameters $L_r = 0.2$ m, $n_e = 5\times 10^{19}\ \mathrm{m}^{-3}$ shows a threshold in the magnetic field around 2 T for $n_r/n_e = 10^{-3}$.

The evolution of the runaway distribution affected by the quasilinear diffusion process associated with WWI has been studied in Ref. 6. As a result of the wave-particle interaction, the particle distribution becomes more isotropic. As the main driving term $\partial f/\partial p_\perp$ becomes smaller, the whistler wave becomes marginally stable ($\gamma_l \lesssim 0$), but as the runaway density increases due to collisional processes, the wave is destabilized again, which leads to further velocity isotropization. If the plasma parameters are such that the whistler wave is destabilized, the time scale of the isotropization is of the

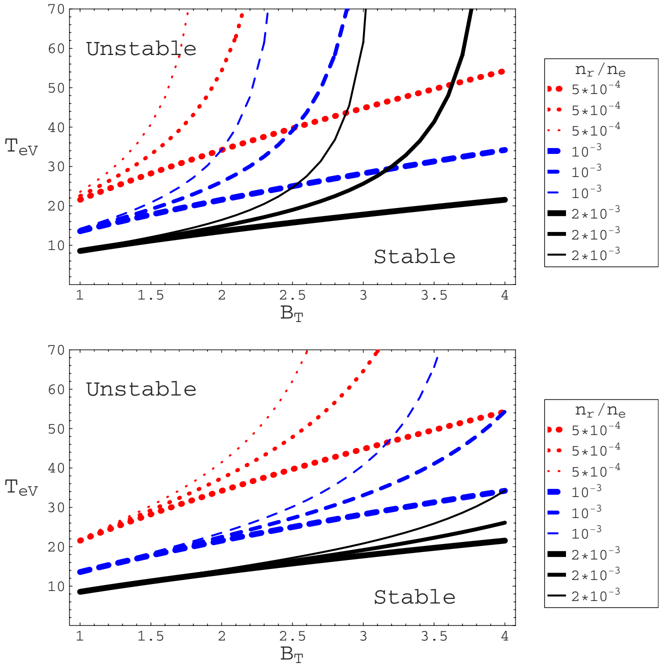

FIG. 1. (Color online) Stability threshold from Eq. (4) for different L_r. The runaway fraction is $n_r/n_e=5\times 10^{-4}$ (dotted), $n_r/n_e=10^{-3}$ (dashed), and $n_r/n_e=2\times 10^{-3}$ (solid). The thickest line is for $L_r=\infty$, the line with middle-thickness is for $L_r=0.4$ m, and the thinnest line is for $L_r=0.2$ m. The upper figure is for $n_e=5\times 10^{19}$ m^{-3} and the lower figure is for $n_e=10^{20}$ m^{-3}.

order of 10^{-5} s. Note that the time scale of the linear and quasilinear evolution of the instability is much shorter than the runaway avalanche growth time. The pitch-angle diffusion leads to higher synchrotron radiation emission[17] that lowers the runaway electron energy and this should lead to a rapid quench of the resonant part of the runaway beam as soon as the runaway density reaches the threshold.

III. CRASH

The magnetic field dependence of runaway generation is introduced via the on-axis current density $j_0=2B/\mu_0 qR$, which is proportional to the magnetic field if the central safety factor q can be assumed to be limited to a value around 1–2 due to operational constraints. A higher on-axis current density leads to higher post-thermal quench electric field, which leads to stronger initial runaway generation by the Dreicer and hot-tail mechanisms. The "seed" produced by these processes is amplified by the secondary avalanche mechanism, the strength of which depends on the total plasma current I_0. The runaway population in tokamaks with large current (e.g., JET, ITER) is mainly produced by the avalanche, but it is very sensitive to the seed runaway density, and therefore also to the magnetic field.

Based on the approximate solution of two coupled differential equations for the runaway electron density and plasma current, a criterion for substantial runaway generation was first derived in Ref. 7 and later refined in Ref. 8. The zero-dimensional model describes the time-dependence of the electric field E induced by the falling current I,

$$E \approx -\frac{L}{2\pi R}\frac{dI}{dt}, \tag{5}$$

where the plasma inductance can be assumed to be $L \simeq \mu_0 R$. The current is the sum of the Ohmic and runaway currents $I=jA_{\text{eff}}=(\sigma E+n_r ec)A_{\text{eff}}$, where j is the on-axis current density, A_{eff} is an effective cross section area of the current, and σ is the conductivity. The criterion takes into account the combined effect of Dreicer and avalanche runaway generation

$$\frac{dn_r}{dt} = \text{Dreicer} + \text{avalanche} \tag{6}$$

due to the rising electric field. From knowing only basic plasma parameters, it is then possible to construct an analytical criterion that estimates whether or not a major fraction of the predisruption plasma current I_0 will be converted to a runaway electron current. A significant runaway current is produced if

$$H = \alpha - \frac{E_D}{4E} - \sqrt{\frac{2E_D}{E}} + \ln\frac{m_e c^2}{T_e} + \frac{11}{8}\ln\frac{E_D}{E} > 0, \tag{7}$$

where $\alpha=(\sqrt{2\pi}/3)(I_0/I_A \ln \Lambda)$ and $I_A=0.017$ MA is the Alfvén current. The postdisruption ratio of the Dreicer field and the parallel electric field, E_D/E, can be obtained from $j_0=\sigma E$ and $j_0=2B/\mu_0 qR$ to be $E_D/E=(3\mu_0 e n_e qR/B)\sqrt{\pi T_e/2m_e}$, where n_e and T_e denote the postdisruption electron density and temperature. Numerical solution of the coupled equations (5) and (6) have confirmed the validity of the criterion given by Eq. (7).[7,8] Note that runaway electron losses, which might play an important role in real disruptions, are not included in this analysis.

A. Hot-tail generation of runaways

The criterion given in Eq. (7) was derived under the assumption that the electron distribution is in a quasisteady state. This assumption is often violated in tokamak disruptions. If the thermal quench duration is less than the collision time at the runaway threshold velocity, then hot-tail generation can be the dominant primary generation process.[9–12] Recent work[13] derived analytical estimates for the amount of hot-tail runaway electrons generated in plasmas with an exponential temperature decrease given by $T=T_0 e^{-t/t_0}$, where T_0 is the initial temperature and t_0 is the thermal quench duration. For a sudden temperature decrease, the hot-tail generated runaways are given by

$$n_h = n_0 \frac{2}{\sqrt{\pi}} u_c e^{-u_c^2}, \tag{8}$$

where

$$u_c^3 = t_0 \nu_0 \left[2 \ln \frac{E_{D0}}{2E_0} - \frac{4}{3} \ln\left(\frac{4}{3} t_0 \nu_0\right) - \frac{5}{3} \right]. \tag{9}$$

In these expressions, $\nu_0 = n_0 e^4 \ln \Lambda / (4\pi\epsilon^2 m_e^2 v_{T0}^3)$ is the initial (predisruption) collision frequency of the thermal electrons, n_0 and v_{T0} are the initial background electron density and thermal speed, and E_{D0}/E_0 is the initial ratio of the Dreicer

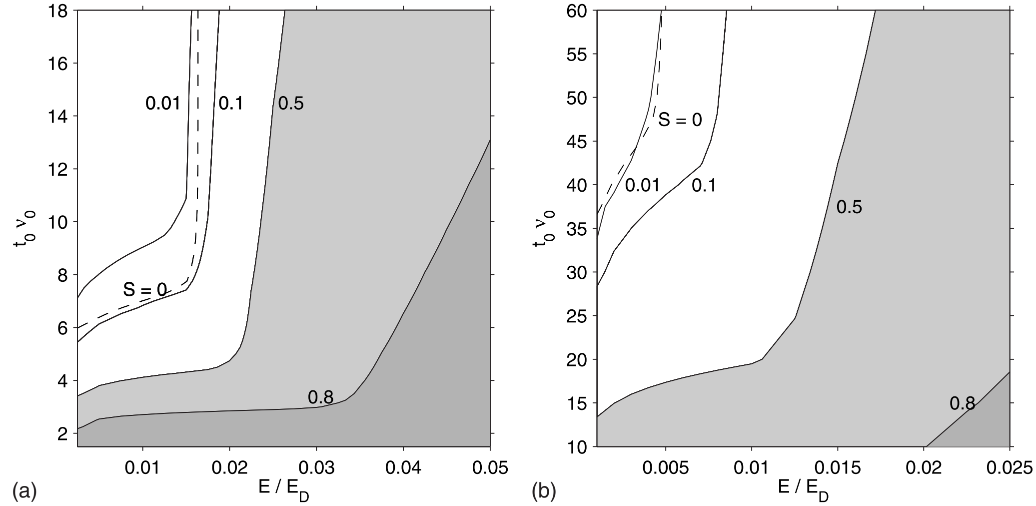

FIG. 2. Efficiency of conversion of thermal current to runaways. In the darkest region, more than 80% of the plasma current is converted into runaway current. The dashed line $S=0$, with S defined by Eq. (11) delineates the region of significant runaway production. (a) JET-like parameters: Initial plasma current $I_0=1.9$ MA, initial current density on axis $j_0=1$ MA/m^2, initial plasma temperature $T_0=3$ keV, and postdisruption temperature $T_e=10$ eV. (b) ITER-like parameters: Plasma current $I_0=15$ MA, current density on axis $j_0=1.5$ MA/m^2, initial plasma temperature $T_0=20$ keV, and postdisruption temperature $T_e=10$ eV.

field and parallel electric field. We assume that the density of seed runaways is the sum of the hot-tail runaway density n_h and the Dreicer generated runaway density [see Eq. (A7) in Ref. 8] $n_D=4\alpha u^2 F(\hat{E})$, where $u^2=T_e/m_e c^2$,

$$F(\hat{E}) = \frac{3\ln\Lambda}{2\sqrt{\pi}}\frac{n_0 ec}{j_0}\frac{1}{u^{15/4}\hat{E}^{3/8}}\exp\left(-\frac{1}{4u^2\hat{E}}-\sqrt{\frac{2}{u^2\hat{E}}}\right), \quad (10)$$

and j_0 is the initial current density. If the runaway current remains a small fraction of the total current, the seed runaway population will be amplified a factor e^α by the avalanche, and the total number of runaways will be $n=(n_D+n_h)e^\alpha$. On the other hand, if the runaway current replaces a large fraction of the Ohmic current, this will cause the electric field to decrease, and the runaway current to saturate before it reaches the initial current I_0. Since this saturation mechanism only comes into play when a considerable runaway current fraction has already been produced, it is not necessary to take it into account in order just to estimate whether or not this will happen. For this, one only needs to determine if $S \equiv \ln n > 0$, so the criterion for large runaway production is

$$S = \alpha + \ln\left[\frac{\sqrt{2}\alpha\ln\Lambda}{\pi}\frac{m_e c^2}{T_e}\left(\frac{E_D}{E}\right)^{11/8} e^{-E_D/4E-\sqrt{2E_D/E}} + \frac{E_D}{E}\sqrt{\frac{m_e c^2}{T_e}}\frac{\sqrt{2}u_c}{3\pi}e^{-u_c^2}\right] > 0, \quad (11)$$

where E/E_D and T_e should be evaluated after the thermal quench. The criterion (7) is recovered in the limit of large $t_0\nu_0$ (so that the last term under the logarithm is negligible) and when the factor $\sqrt{2}\alpha\ln\Lambda/\pi$ multiplying the first term under the logarithm is neglected. Due to the weak dependence of the hot tail generation on E_D/E compared with the dependence on $\nu_0 t_0$, the B_T dependence of S comes mainly from the Dreicer mechanism. The avalanche term α depends only on I_0, which we choose to vary independently of B_T. Note that to keep q and I_0 constant while varying B_T (or equivalently j_0) corresponds to varying the current cross section A_{eff}.

Figure 2 shows the fraction of the plasma current that is converted into runaways as a function of $\nu_0 t_0$ and the normalized electric field after a thermal quench. The runaway current evolution was here calculated using the zero-dimensional model in Eqs. (5) and (6), extended to also include hot tail generation according to the velocity moment method described in Ref. 13. For short thermal quenches or large electric fields the number of hot-tail and Dreicer runaways is substantial and most of the thermal current will be converted into runaways. The CRASH line, shown as a dashed line in the figure, gives a good approximation for the region of significant runaway production. The vertical part of the CRASH line is due to the Dreicer generation and the horizontal part is due to hot-tail generation.

IV. DISCUSSION

Similar to the WWI threshold, CRASH shows that runaway generation is expected only for magnetic fields above a certain threshold, which depends on the initial plasma current and the electron density and temperature. Approximately, for JET-like values ($q=1.5$, $R=3$ m, $I_0=2$ MA), when hot-tail generation is not very strong, Eq. (11) can be written as

$$B_T > n_{19}\sqrt{T_{\mathrm{eV}}/I_{\mathrm{MA}}}/4, \quad (12)$$

where I_{MA} is the initial plasma current in mega-amperes. For ITER-like values ($q=1.5$, $R=6$ m, $I_0=15$ MA), the hot-tail

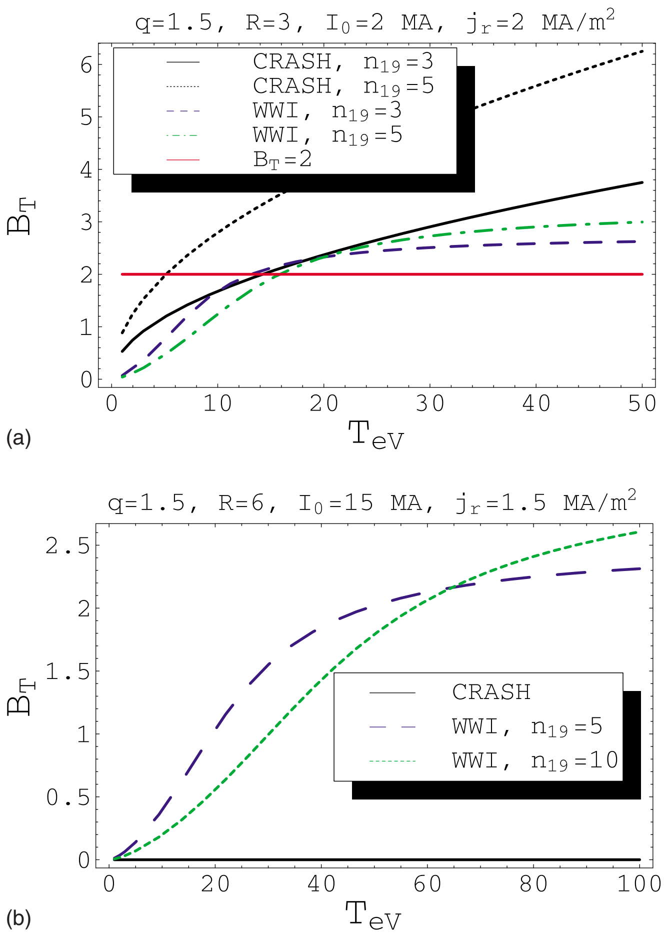

FIG. 3. (Color online) Critical magnetic field for significant runaway generation as a function of T_{eV} for different electron densities. (a) JET-like parameters: $q=1.5$, $R=3$ m, $I_0=2$ MA, $\nu_0 t_0=10$ (for CRASH), $L_r=0.2$ m, $j_r=2$ MA/m^2 (for WWI). (b) ITER-like parameters: $q=1.5$, $R=6$ m, $I_0=15$ MA $\nu_0 t_0<5$ (for CRASH), $L_r=0.3$ m, $j_r=1.5$ MA/m^2 (for WWI).

generation is dominant and S in Eq. (11) is always positive, unless $\nu_0 t_0$ is very large. This means that CRASH does not lead to a threshold in the magnetic field for ITER-like parameters.

The effective beam radius, which determines the magnitude of the convective damping term in the WWI threshold (4), can be calculated by solving the one-dimensional counterparts of the coupled Eqs. (5) and (6) as has been done in Ref. 8 and calculating the runaway current density profile. In Fig. 5(a) of Ref. 8 this has been calculated for various postdisruption temperatures for a JET-like plasma, and from these calculations L_r can be estimated to be 0.2 m. In a numerical simulation for an ITER-scenario consistent with the parameters given in Fig. 3(b), the value for L_r was around 0.3 m at the time when the runaway current density on axis was $j_r=1.5$ MA/m^2 (same as the initial Ohmic current density j_0). The dependence of L_r on the cooling time and the final postdisruption temperature can be seen in Table I, showing the radius of the runaway beam defined as $L_r(t)=\sqrt{I_r/[\pi j_r(t)]}$. In the third column of the table L_r is given at a time $t_\star$ when $j_r(t_\star)=j_0=1.5$ MA/m^2. The beam width varies considerably with time, as can be seen by comparing with the maximum of $L_r(t)$ in the fourth column. In our study we only take a constant value for L_r, whereas in reality the evolution of the width and strength of the runaway beam is dynamically coupled to the instability as can be seen in Fig. 1.

TABLE I. Radius of the runaway beam for different postdisruption temperatures and thermal quench times.

T_{final} (eV)	t_0 (ms)	$L_r(t_\star)$ (m)	max L_r (m)
10	5	0.32	0.75
20	5	0.29	0.65
30	5	0.27	0.59
50	5	0.24	0.51
30	10	0.13	0.30
30	15	0.09	0.26

Figure 3 compares the magnetic field determined by WWI and CRASH for JET-like and ITER-like parameters. Above the CRASH lines $S>0$ and therefore substantial runaway generation is expected. Above the WWI lines whistler waves are stable and a runaway beam can form. For temperatures less than ~10 eV, the magnetic field threshold for stability of whistler waves is very low, and therefore the WWI will not stop the runaway beam formation. But for temperatures above 10 eV, as the convective damping becomes comparable to collisional damping, the temperature dependence becomes less important and the WWI leads to a threshold around 2 T for both JET-like and ITER-like parameters. Due to the strong magnetic field dependence of the convective damping, the threshold for WWI turns out to be only weakly dependent of the other plasma parameters. As mentioned in Sec. II a more exact threshold for the destabilization can be obtained by numerical simulations described in Ref. 6, and this would shift the WWI curves toward about 20%–30% lower temperatures. Experimentally, the postdisruption temperature is uncertain, but it is believed to be around 10 eV.

In the JET-like case CRASH leads to a higher magnetic field threshold than WWI. This means that the zero-dimensional model presented in Sec. III does not predict substantial runaway generation below the magnetic field indicated by the solid and dotted lines for the chosen cooling time. Therefore one may draw the conclusion that there is no runaway beam for the WWI to stop. However, if the cooling time is shorter, CRASH leads to a lower magnetic field threshold, and below $t_0 \simeq 1$ ms (for $q=1.5$, $R=3$ m, $I_0=2$ MA, $T_0=3$ keV, $n_{19}=3$, $B_T=2$), S is always positive. Note that CRASH depends approximately linearly on the electron density, and the probability for runaway production is higher for low density. In ITER-like disruptions, the thermal quench time is predicted to be $t_0=1-10$ ms,[18] so that $\nu_0 t_0<5$ and runaways are always likely to be produced due to the hot-tail generation, although the beam formation will probably be stopped by WWI below 2 T.

We have demonstrated that the WWI is an effective loss mechanism for secondary runaways. There are several other processes that can limit the runaway energy or cause loss of runaways, such as, synchrotron radiation,[17] Bremsstrahlung,[19] unconfined drift orbit losses,[20] resonance between gyromotion and magnetic field ripple,[21] and radial diffusion due to magnetic field fluctuations.[22] Since these mechanisms are not considered in this work, the results presented here are expected to give a lower limit of the magnetic field threshold for runaway production.

V. CONCLUSIONS

For a given temperature, density, and runaway fraction, if the magnetic field is below a critical value, the whistler wave can be destabilized by relativistic secondary runaway electrons. This mechanism offers a possible explanation for the magnetic field threshold for runaway generation observed in tokamak disruptions. Lower runaway fractions are needed for destabilization in plasmas with high temperature, since the collisional damping is lower. The convective damping due to the localization of the runaway beam can be of the same order of magnitude as the collisional damping for high temperature and strong magnetic field. The convective damping is sensitive to the radius of the runaway beam and the fraction of runaways in the plasma n_r/n_e, and these depend on the other plasma parameters, for instance, the final temperature and the cooling time. Therefore, runaway production and suppression by WWI is a dynamical process, in which the runaways trigger the WWI. The consequent scattering affects the strength and width of the runaway beam, which in turn affects the damping of the WWI. It is difficult to predict exactly where the threshold in B might be without self-consistent simulations of the runaway distribution function and electric field evolution, that could be achieved for instance by the ARENA code (described in Ref. 23), coupled to an evaluation of the instability growth rate.

The magnetic field dependence of the runaway production has been studied by considering the coupled dynamics of the runaway generation and evolution of plasma current, including the hot-tail generation of runaways. This leads to an analytical criterion for runaway avalanche (CRASH) that can be used to estimate if there will be a substantial runaway generation or not. CRASH can be shown to lead to a magnetic field threshold unless hot-tail generation dominates. But in rapidly cooling plasmas, where hot-tail generation gives rise to a substantial runaway population, the whistler waves can stop the runaway formation at a certain magnetic field unless the postdisruption temperature is too low. If the postdisruption temperature is very low then whistler waves are stable and WWI cannot stop the runaway beam formation.

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[1] R. Aymar, P. Barabaschi, and Y. Shimomura, Plasma Phys. Controlled Fusion **44**, 519 (2002).
[2] R. D. Gill, B. Alper, M. de Baar, T. C. Hender, M. F. Johnson, V. Riccardo, and Contributors to the EFDA-JET Workprogramme, Nucl. Fusion **42**, 1039 (2002).
[3] R. Yoshino, S. Tokuda, and Y. Kawano, Nucl. Fusion **39**, 151 (1999).
[4] V. Riccardo, Plasma Phys. Controlled Fusion **45**, A269 (2003).
[5] T. Fülöp, G. Pokol, P. Helander, and M. Lisak, Phys. Plasmas **13**, 062506 (2006).
[6] G. Pokol, T. Fülöp, and M. Lisak, Plasma Phys. Controlled Fusion **50**, 045003 (2008).
[7] P. Helander, L.-G. Eriksson, and F. Andersson, Plasma Phys. Controlled Fusion **44**, B247 (2002).
[8] H. M. Smith, P. Helander, L.-G. Eriksson, D. Anderson, M. Lisak, and F. Andersson, Phys. Plasmas **13**, 102502 (2006).
[9] R. W. Harvey, V. S. Chan, S. C. Chiu, T. E. Evans, M. N. Rosenbluth, and D. G. Whyte, Phys. Plasmas **7**, 4590 (2000).
[10] S. C. Chiu, M. N. Rosenbluth, R. W. Harvey, and V. S. Chan, Nucl. Fusion **38**, 1711 (1998).
[11] P. Helander, H. Smith, T. Fülöp, and L.-G. Eriksson, Phys. Plasmas **11**, 5704 (2004).
[12] H. M. Smith, P. Helander, and T. Fülöp, Phys. Plasmas **12**, 122505 (2005).
[13] H. M. Smith and E. Verwichte, Phys. Plasmas **15**, 072502 (2008).
[14] P. H. Rebut, Nucl. Fusion **25**, 1011 (1985).
[15] M. N. Rosenbluth and S. V. Putvinski, Nucl. Fusion **37**, 1355 (1997).
[16] M. Brambilla, Phys. Plasmas **2**, 1094 (1995).
[17] F. Andersson, P. Helander, and L.-G. Eriksson, Phys. Plasmas **8**, 5221 (2001).
[18] T. C. Hender, J. C Wesley, J. Bialek, A. Bondeson, A. H. Boozer, R. J. Buttery, A. Garofalo, T. P. Goodman, R. S. Granetz, Y. Gribov, O. Gruber, M. Gryaznevich, G. Giruzzi, S. Günter, N. Hayashi, P. Helander, C. C. Hegna, D. F. Howell, D. A. Humphreys, G. T. A. Huysmans, A. W. Hyatt, A. Isayama, S. C. Jardin, Y. Kawano, A. Kellman, C. Kessel, H. R. Koslowski, R. J. La Haye, E. Lazzaro, Y. Q. Liu, V. Lukash, J. Manickam, S. Medvedev, V. Mertens, S. V. Mirnov, Y. Nakamura, G. Navratil, M. Okabayashi, T. Ozeki, R. Paccagnella, G. Pautasso, F. Porcelli, V. D. Pustovitov, V. Riccardo, M. Sato, O. Sauter, M. J. Schaffer, M. Shimada, P. Sonato, E. J. Strait, M. Sugihara, M. Takechi, A. D. Turnbull, E. Westerhof, D. G. Whyte, R. Yoshino, H. Zohm, the ITPA MHD, and Disruption and Magnetic Control Topical Group, Nucl. Fusion **47**, S166 (2007).
[19] M. Bakhtiari, G. J. Kramer, M. Takechi, H. Tamai, Y. Miura, Y. Kusama, and Y. Kamada, Phys. Rev. Lett. **94**, 215003 (2005).
[20] H. Knoepfel and D. A. Spong, Nucl. Fusion **19**, 785 (1979).
[21] B. Kurzan, K. H. Steuer, and G. Fussmann, Phys. Rev. Lett. **75**, 4626 (1995).
[22] P. Helander, L.-G. Eriksson, and F. Andersson, Phys. Plasmas **7**, 4106 (2000).
[23] L.-G. Eriksson, P. Helander, F. Andersson, D. Anderson, and M. Lisak, Phys. Rev. Lett. **92**, 205004 (2004).