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A unified treatment of kinetic effects in a tokamak pedestal

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Abstract
We consider the effects of a finite pedestal radial electric field on ion orbits using a unified approach. We then employ these modified orbit results to retain finite $\boldsymbol{E}\times\boldsymbol{B}$ drift departures from flux surfaces in an improved drift-kinetic equation. The procedure allows us to make a clear distinction between transit averages and flux surface averages when solving this kinetic equation. The technique outlined here is intended to clarify and unify recent evaluations of the banana regime decrease and plateau regime alterations in the ion heat diffusivity; the reduction and possible reversal of the poloidal flow in the banana regime, and its augmentation in the plateau regime; the increase in the bootstrap current; and the enhancement of the residual zonal flow regulation of turbulence.

1. Introduction

It is standard in tokamak kinetic theory to assume that the poloidal ion gyroradius is small compared with all macroscopic scale lengths, but in the pedestal this approximation can break down. This challenge is addressed by our recently developed techniques that account for the presence of short radial scale lengths in a subsonic pedestal. Using these techniques, we have calculated both the banana [1, 2] and plateau [3] regime modifications to the neoclassical ion heat flux, ion and impurity flows, and the bootstrap current; and extended the residual zonal flow calculation to the pedestal case [4, 5]. Here, we present a general formalism providing deeper insight into these detailed calculations by considering the effects of the finite pedestal radial electric field $\boldsymbol{E}$ on the ion orbits by an improved procedure. This unified approach highlights the intricacies of the previous investigations, thereby forming a firm basis for how these kinetic calculations are best performed. In particular, we present a general method of treating finite electric field effects on ion orbits that allows us to solve a reduced kinetic equation for the pedestal modifications to the results of the usual evaluations of neoclassical transport in the banana and plateau regimes [6–8], and the residual zonal flow calculation [9, 10]. The emphasis herein is on the steps that differ from the usual neoclassical and residual zonal flow treatments. The basis of the technique is the convenience of employing the canonical angular momentum as the radial gyrokinetic variable [11]. This choice allows us to perform calculations in the pedestal by accounting for the presence of strong radial density (and electron temperature) gradient scale length on the order of a poloidal ion gyroradius $\rho_{\mathrm{pi}} = \rho_{\mathrm{i}} B/B_{\mathrm{p}}$, where $B/B_{\mathrm{p}} \gg 1$ and $\rho_{\mathrm{i}} = v_{\mathrm{i}}/\Omega_{\mathrm{i}}$ is the ion gyroradius, with $v_{\mathrm{i}} = (2T_{\mathrm{i}}/M)^{1/2}$ the ion thermal speed and $\Omega_{\mathrm{i}} = ZeB/Mc$ the ion gyrofrequency for ions of mass M, charge number Z and temperature T_{i}.

In a subsonic pedestal the $\boldsymbol{E} \times \boldsymbol{B}$ drift and the ion diamagnetic drift must cancel to lowest order [11–13]. In the helium discharges on DIII-D this behavior is verified since the background ion temperature can be measured directly [14]. In this situation the ions are electrostatically confined to lowest order and the associated radial electric field is so large that the $\boldsymbol{E} \times \boldsymbol{B}$ drift velocity can compete with the poloidal component of the parallel streaming to modify trajectories. These modifications introduce electric field dependence into the neoclassical ion and impurity flows via the usual ion temperature gradient term [1, 2]. The ion temperature pedestal is always at least B/B_{p} wider than ρ_{pi} because of the constraint that the entropy production must vanish in a banana or plateau regime pedestal [11]. In addition, the ion heat flux [1, 3] and the bootstrap current [2, 3] are altered, along with the residual zonal flow [4, 5] response of Rosenbluth and Hinton [9]. In spite of these changes the plasma remains intrinsically ambipolar [15, 16] in the more general sense that $\boldsymbol{E}$ (or more precisely, the difference between the $\boldsymbol{E} \times \boldsymbol{B}$ and ion diamagnetic drifts) remains undetermined until conservation of toroidal angular momentum is solved, but unlike the core the heat flux depends on the radial electric field as does the ambipolar particle flux.

Sonic flow in a pedestal with a poloidal gyroradius width requires the $\boldsymbol{E} \times \boldsymbol{B}$ drift to be comparable to the ion thermal speed v_{i} and that the $\boldsymbol{E} \times \boldsymbol{B}$ and ion diamagnetic drift do not cancel to lowest order when $B_{\mathrm{p}} \ll B$. Sonic flows are also possible when the $\boldsymbol{E} \times \boldsymbol{B}$ drift is on the order of v_{i} and much larger than diamagnetic flows. For sonic pedestal flow the treatment here must be altered to retain centrifugal effects.

In section 2 we present a unified approach to retaining the effect of the radial electric field on the ion orbits to be used for both the banana and plateau regime cases. The potential is taken to be a quadratic function of flux, so our analysis captures finite orbit effects due to both $\boldsymbol{E}$ itself and the variation of $\boldsymbol{E}$ across the pedestal, by keeping the distinction between flux and drift surfaces. A drift-kinetic equation valid for treating neoclassical transport in both regimes of collisionality, which also retains the zonal flow residual is efficiently derived in section 3. The generality of the results in these two sections then allows us to conveniently derive all earlier neoclassical banana [1, 2] and plateau [3] results in section 4, as well as all previous results for the radial electric field modified zonal flow residual [4] and its orbit squeezing generalization [5] in section 5. Moreover, the treatment of collisions is streamlined so that the transit averaged collisional constraints are performed retaining finite drift orbit effects.

2. Radial electric field effects on the treatment of ion orbits

When the radial density scale length of the pedestal becomes comparable to the poloidal ion gyroradius, ρ_{pi}, the ion flow speed can only be subsonic if the $\boldsymbol{E} \times \boldsymbol{B}$ and ion diamagnetic flows cancel to lowest order. The lowest order cancellation means that the ions are electrostatically confined with the radial electric field satisfying $(Ze/T_{\mathrm{i}})\mathrm{d}\Phi/\mathrm{d}r \approx -\mathrm{d}\ln p_{\mathrm{i}}/\mathrm{d}r \sim 1/\rho_{\mathrm{pi}} \gg 1/a$, with a the minor radius [11]. Such a strong radial electric field results in an $\boldsymbol{E} \times \boldsymbol{B}$ drift that competes with the poloidal component of parallel streaming, while staying well below $v_{\rm i}$, since $c\boldsymbol{E} \times \boldsymbol{B}/B^2 \sim (B_{\rm p}/B)v_{\rm i}$, where $\boldsymbol{B} = I\nabla\zeta + \nabla\zeta \times \nabla\psi$ with ζ, ϑ and ψ the toroidal angle, poloidal angle and poloidal flux variables, respectively, $B = |\boldsymbol{B}|$ and $|\nabla\psi| = RB_{\rm p}$. For a $\mathrm{d}\Phi/\mathrm{d}r$ this large, the variation in potential energy over an orbit width is sufficient to make electrostatic trapping important, even when the potential is a flux function [1–5, 11]. In addition to introducing this finite orbit effect, orbit squeezing can enter [17]. The competition between the $\boldsymbol{E} \times \boldsymbol{B}$ drift and the poloidal projection of parallel streaming makes it necessary to retain the distinction between the surfaces of constant magnetic flux and the drift surfaces on which the canonical angular momentum remains constant. However, to obtain analytical results we must assume the inverse aspect ratio ε is small compared with unity ($\varepsilon \ll 1$). With this assumption the transport remains local because the trapped and barely passing ion orbit widths are of order $\varepsilon^{1/2}\rho_{\rm pi}$, and so are less than the equilibrium pedestal scale length that is allowed to be as small as $\rho_{\rm pi}$.

To see how the orbits are modified we employ the magnetic moment $\mu = v_\perp^2/2B$, along with the drift approximation to the canonical angular momentum constant of the motion ψ_* defined for $B_{\rm p}/B \ll 1$ by

$$\psi_* \equiv \psi - (Mc/Ze)R^2\vec{v}\cdot\nabla\zeta \approx \psi - (Iv_\parallel/\Omega_{\rm i}), \tag{1}$$

and the notationally convenient pseudo-kinetic energy variable E_* defined by

$$E_* \equiv E - (Ze/M)\Phi(\psi_*) = v^2/2 + (Ze/M)[\Phi(\psi) - \Phi(\psi_*)], \tag{2}$$

which is a constant of the motion since it only depends on ψ_* and the total energy constant of the motion $E = v^2/2 + (Ze/M)\Phi(\psi)$. We also assume the potential only depends on ψ with a quadratic dependence so that using (1) gives

$$\Phi(\psi) = \Phi(\psi_*) + (Iv_\parallel/\Omega_{\rm i})\Phi'(\psi_*) + (1/2)(Iv_\parallel/\Omega_{\rm i})^2\Phi'', \tag{3}$$

with Φ'' a constant. The utility of these variables will become clearer in the next section when they are used to rewrite the drift-kinetic equation. In this section we obtain the relations needed in subsequent sections.

We define the effective $\boldsymbol{E} \times B$ velocities in the poloidal magnetic field and the orbit squeezing factor S as

$$u = cI\Phi'/B, \qquad u_* = cI\Phi'_*/B \qquad \text{and } S = 1 + (cI^2\Phi''/\Omega_{\rm i}B), \tag{4}$$

with $\Phi'(\psi_*) = \Phi'_*$ and $\Phi'(\psi) = \Phi'$. Note that the only difference between u_* and u is whether the potential derivative is a function of ψ_* or ψ, respectively, since both u_* and u depend on B, with $B(\psi_*) \approx B(\psi)$ to the order we require. We then use $\Phi'(\psi) = \Phi'(\psi_*) + (\psi - \psi_*)\Phi''$ to obtain the useful and important relation

$$v_{||} + u = Sv_{||} + u_*. \tag{5}$$

In addition, we can use definitions (4) and the preceding result to rewrite E_* as

$$\begin{aligned} E_* &\equiv Sv_\parallel^2/2 + \mu B + u_*v_\parallel = [(Sv_\parallel + u_*)^2/2S] + \mu B - u_*^2/2S \\ &= [(v_\parallel + u)^2/2S] + \mu B - u_*^2/2S. \end{aligned} \tag{6}$$

Solving (6) for $v_\parallel$ gives $v_\parallel + u = Sv_\parallel + u_* = \pm(2SE_* + u_*^2 - 2S\mu B)^{1/2}$ so $sgn(v_\parallel + u) = sgn(Sv_\parallel + u_*)$ must also be specified when ψ_*, ϑ, E_*, μ are used as the variables. Unlike the more familiar situation with ψ, ϑ, E, μ as the variables, $sgn(v_\parallel)$ is no longer a useful coordinate since two phase space locations in ψ_*, ϑ, E_*, μ can have the same sign. Phase space remains split into trapped and passing regions defined by the preceding radicand at $\vartheta = \pi$. The trapped distribution function must be independent of $sgn(Sv_\parallel + u_*)$ at the bounce points where it vanishes. This property is used in section 4 to define the new transit average annihilation operation holding fixed ψ_*. The preceding results are valid for arbitrary aspect ratio.

To make further progress it is necessary to introduce the inverse aspect ratio expansion of the magnetic field by letting

$$B_0/B = 1 - 2\varepsilon\ \sin^2(\vartheta/2) = 1 - \varepsilon + \varepsilon\ \cos\ \vartheta, \tag{7}$$

where B_0 is the magnetic field at $\vartheta = 0$. Retaining inverse aspect ratio corrections through order ε we may then write (6) as

$$(v_\parallel + u)^2/2 = (Sv_\parallel + u_*)^2/2 = W(1 - \Lambda B/B_0) = (1 - \Lambda)W[1 - \kappa^2\ \sin^2(\vartheta/2)], \tag{8}$$

where W, Λ and κ^2 are adiabatic invariants to order ε defined by

$$W = S_0E_* + 2(S_0 - 1)(E_* - \mu B_0) + (3/2)u_{*0}^2, \tag{9}$$

$$\Lambda = \frac{\kappa^2}{\kappa^2 + 2\varepsilon} = \frac{S_0\mu B_0 + 2(S_0 - 1)(E_* - \mu B_0) + u_{*0}^2}{S_0E_* + 2(S_0 - 1)(E_* - \mu B_0) + (3/2)u_{*0}^2}, \tag{10}$$

$$\kappa^2 = \frac{2\varepsilon\Lambda}{1 - \Lambda} = \frac{2\varepsilon[S_0\mu B_0 + 2(S_0 - 1)(E_* - \mu B_0) + u_{*0}^2]}{S_0(E_* - \mu B_0) + (1/2)u_{*0}^2} \tag{11}$$

with

$$u_{*0} = cI\Phi_*'/B_0, \qquad S_0 = 1 + (cI^2\Phi''/\Omega_0B_0), \tag{12}$$

and $\Omega_0 = ZeB_0/Mc$. The new variables W and Λ reduce to the familiar ones $v^2/2$ and $\lambda = 2\mu B_0/v^2$ in the limit of vanishing radial electric field ($u_{*0} = 0$ and $S_0 = 1$). Equation (8) then reduces to the usual result that $v_\parallel^2 = v^2(1 - \lambda B/B_0)$. Equations (9)–(11) are consistent with the expressions from [1–5] written in terms of $v_{\parallel 0}$, with the parallel velocity evaluated at $\vartheta = 0$. The trapped-passing boundary occurs at $\kappa^2 = 1$ or $\Lambda = 1/(1 + 2\varepsilon)$, and is shifted and distorted from the usual boundary as noted in [1, 4, 5]. Equations (8)–(11) are particularly convenient for switching between ψ_* and ψ variables. In the next section they will allow the transit averages that follow an ion trajectory to be performed holding ψ_* fixed, while evaluating the flux surface averages at fixed ψ. As usual, our analysis only accurately retains order $\varepsilon^{1/2}$ corrections; ε corrections to transport coefficients and the residual zonal flow are ignored.

3. Drift-kinetic equation

Using ψ_*, ϑ, E_* and μ as our independent variables for an axisymmetric tokamak results in the drift-kinetic equation $\partial f/\partial t + \dot{\psi}_*\partial f/\partial\psi_* + \dot{\vartheta}\partial f/\partial\vartheta + \dot{E}_*\partial f/\partial E_* = C\{f\}$ since μ is an adiabatic invariant. Here $f = f(\psi_*, \vartheta, E_*, \mu)$ is the gyroaveraged ion distribution function and C is the Fokker–Planck collision operator for ion–ion collisions. Using $\dot{\psi}_* = 0$, allowing slow time dependence so that $\dot{E}_* = (Ze/M)\partial\Phi/\partial t$, and working to high enough order to retain $\boldsymbol{E} \times \boldsymbol{B}$ plus magnetic drifts, $\boldsymbol{v}_\mathrm{d}$, gives $\dot{\vartheta} = (v_\parallel\boldsymbol{n} + \boldsymbol{v}_\mathrm{d}) \cdot \nabla\vartheta$ with $\boldsymbol{n} = \boldsymbol{B}/B$. As a result, our drift-kinetic equation is simply

$$\partial f/\partial t + (v_\parallel\boldsymbol{n} + \boldsymbol{v}_\mathrm{d}) \cdot \nabla\vartheta\,\partial f/\partial\vartheta + (Ze/M)(\partial\Phi/\partial t)\partial f/\partial E_* = C\{f\}. \tag{13}$$

By retaining slow time dependence, (13) allows us to consider the residual zonal flow problem and neoclassical transport in the pedestal in an additive fashion. To do so we let

$$f = f_*(\psi_*, E) + h(\psi_*, \vartheta, E_*, \mu, t) \tag{14}$$

where

$$f_* = f_*(\psi_*, E) = \eta(\psi_*)[M/2\pi T_\mathrm{i}(\psi_*)]^{3/2}\exp[-ME/T_\mathrm{i}(\psi_*)], \tag{15}$$

with both the pseudo density, $\eta(\psi) = n_i(\psi)\exp[Ze\Phi(\psi)/T_i(\psi)]$, and the ion temperature, $T_i(\psi)$, weakly varying functions of space in the pedestal, as required to make the entropy production vanish [11]. Then f_* makes the Vlasov operator vanish and can be Taylor expanded about ψ. Note that the ion density n_i and the electrostatic potential are allowed to be strong functions of ψ so the density cannot be expanded about ψ. The background potential $\Phi(\psi)$ is assumed quadratic in ψ, while the fluctuating potential is assumed to be a small correction that does not impact ion orbits. Expanding (15) about the Maxwellian

$$f_M = f_M(\psi, E) = \eta(\psi)[M/2\pi T_i(\psi)]^{3/2}\exp[-ME/T_i(\psi)] \tag{16}$$

gives

$$f_* = f_M\left\{1 - \frac{Iv_\parallel}{\Omega_i}\left[\frac{1}{p_i}\frac{\partial p_i}{\partial\psi} + \frac{Ze}{T_i}\frac{\partial\Phi}{\partial\psi} + \left(\frac{Mv^2}{2T_i} - \frac{5}{2}\right)\frac{1}{T_i}\frac{\partial T_i}{\partial\psi}\right] + \cdots\right\}, \tag{17}$$

where $T_i\,\mathrm{d}p_i/p_i\,\mathrm{d}T_i \sim L_T/\rho_{pi} \sim B/B_p$, with L_T the ion temperature scale length. Note that the correction to f_M is $f_* - f_M + h$. Then the kinetic equation (13) to the requisite order becomes

$$\partial h/\partial t + (v_\parallel + u)\boldsymbol{n}\cdot\nabla\vartheta\,\partial h/\partial\vartheta + (Ze/M)(\partial\Phi/\partial t)\partial f_M/\partial E = C_\ell\{f - f_M\}, \tag{18}$$

where we have allowed for the possibility of finite $\boldsymbol{E}\times\boldsymbol{B}$ effects by retaining the u term from $\dot{\vartheta}$, and use C_ℓ to denote the linearized ion–ion collision operator. For the residual zonal flow calculation collisions are neglected. For the neoclassical transport calculations considered next, the time derivatives are dropped. The linearity of (18) means that we can add the neoclassical and zonal flow contributions to the perturbed ion distribution function that is found in the next two sections.

4. Neoclassical transport

Dropping the time derivatives and noting that for the linearized ion–ion collision operator $C_\ell\{(1, \boldsymbol{v}, v^2/2)f_M\} = 0$, it is convenient to define

$$H = h - f_M\frac{Iv_\parallel}{\Omega_i T_i}\left(\frac{Mv^2}{2T_i} - \frac{5}{2} + \frac{B^2 k}{2\langle B^2\rangle}\right)\frac{\partial T_i}{\partial\psi}, \tag{19}$$

where k is a flux function to be determined. This definition is convenient because H turns out to be the only part of the distribution function that gives radial transport and poloidal flow. The $\partial p_i/\partial\psi$ and $\partial\Phi/\partial\psi$ terms in (17) do not contribute to the right side of (13) as transport drive terms—only the $\partial T_i/\partial\psi$ term does. The k term is added for later use to restore the $C_\ell\{v_\parallel f_M\} = 0$ property when it becomes necessary to employ an approximate collision operator to obtain explicit results. The B dependence of the k coefficient is made explicit to obtain a form for the flows that will be divergence-free, where $\langle B^2\rangle = B_0^2[1+O(\varepsilon)] \approx B_0^2$ with $\langle\ldots\rangle$ denoting a flux surface average, and order ε corrections to the k term in (19) unimportant. Using (19) in the steady-state version of (18) simplifies it to

$$(v_\parallel + u)\boldsymbol{n}\cdot\nabla\vartheta\,\partial h/\partial\vartheta = C_\ell\{H\}; \tag{20}$$

the form for the kinetic equation that is useful in both the banana and plateau regimes.

4.1. Plateau regime

The plateau regime calculation is performed in detail in [3] and easily recovered from the preceding formalism. The procedure is to first realize that because of the singular nature of H at $v_\parallel + u = 0$, (20) is in the correct form to make the usual plateau replacement of $C_\ell\{H\}$

by $-\nu H$ to resolve the singularity. The singularity arises because the ϑ dependence of $h - H$ from B and $v_\parallel$ in (19) acts as a $\sin\vartheta$ drive that leads to a singularity at $v_\parallel + u = 0$ for $\nu = 0$. We then also use (5) to write the streaming operator in terms of the ψ_*, ϑ, E_* and μ. Once the equation is solved for h the solution must be changed back to the ψ, ϑ, E and μ variables to perform the velocity space integrals holding ψ fixed (these details of the procedure are addressed in the next subsection). The flux function k is determined by requiring that the ions give no particle transport as required by the momentum conservation property $C_\ell\{v_\parallel f_M\} = 0$ of the full linearized ion–ion collision operator. The corrected result [3] is

$$k = A_p(U^2) = \frac{1 + 4U^2 + 6U^4 + 12U^6}{1 + 2U^2 + 2U^4}, \tag{21}$$

with $\boldsymbol{n} \cdot \nabla\vartheta = qR_0$, q the safety factor, R_0 the major radius at $\vartheta = 0$ and

$$U = \frac{cI}{v_i B_0} \frac{\partial \Phi}{\partial \psi}. \tag{22}$$

When these steps are carried out, orbit squeezing does not enter and the following results are obtained for the parallel ion flow $V_{\parallel i} = n_i^{-1} \int d^3 v v_\parallel f$, radial ion heat flux $\langle \boldsymbol{q}_i \cdot \nabla\psi \rangle = \langle \int d^3 v (Mv^2/2) f \boldsymbol{v}_d \cdot \nabla\psi \rangle$ and bootstrap current J_{bs} in the plateau regime (denoted by an index p):

$$n_i V_{\parallel i}^p = -\frac{Ip_i}{M\Omega_i} \left[\frac{1}{p_i} \frac{\partial p_i}{\partial \psi} + \frac{Ze}{T_i} \frac{\partial \Phi}{\partial \psi} + \frac{A_p(U^2)B^2}{2\langle B^2 \rangle T_i} \frac{\partial T_i}{\partial \psi} \right], \tag{23}$$

$$\langle \boldsymbol{q}_i^p \cdot \nabla\psi \rangle = -3\sqrt{\frac{\pi}{2}} \frac{\varepsilon^2 I^2 n_i}{qR_0\Omega_i^2} \left(\frac{T_i}{M}\right)^{3/2} \frac{\partial T_i}{\partial \psi} L_p(U^2) \tag{24}$$

and

$$J_{bs}^p = -\sqrt{\frac{\pi}{2}} \frac{\varepsilon^2 cIBv_e}{qR_0\nu_e\langle B^2 \rangle} \frac{\sqrt{2} + 4Z}{Z(2 + \sqrt{2}Z)} \left[\frac{\partial p}{\partial \psi} + \frac{(\sqrt{2} + 13Z)n_e}{2(\sqrt{2} + 4Z)} \frac{\partial T_e}{\partial \psi} + \frac{A_p(U^2)n_e}{2Z} \frac{\partial T_i}{\partial \psi} \right], \tag{25}$$

where $p_e = n_e T_e$, $v_e = (2T_e/m)^{1/2}$, $\nu_e = 4(2\pi)^{1/2} n_e e^4 \ln\Lambda/(3m^{1/2}T_e^{3/2})$, A_p is given by (21), and

$$L_p(U^2) = \frac{1 + 4U^2 + 8U^4 + 4U^6(4 + U^2)/3}{1 + 2U^2 + 2U^4} \exp(-U^2). \tag{26}$$

The poloidal flow of Pfirsch–Schluter trace impurities (subscript z) for plateau regime background ions is then given by

$$V_z^p\big|_{pol} = -\frac{cIB_pT_i}{Ze\langle B^2 \rangle} \left[\frac{1}{p_i} \frac{\partial p_i}{\partial \psi} - \frac{ZT_z}{Z_zT_ip_z} \frac{\partial p_z}{\partial \psi} + \frac{A_p(U^2)}{2T_i} \frac{\partial T_i}{\partial \psi} \right]. \tag{27}$$

The factors A_p and L_p account for the modifications of the usual plateau results [8] by finite radial electric field effects and equal one at $U = 0$. The bootstrap current is evaluated by a two-term Laguerre expansion in combination with an adjoint method [3]. The function A_p is a monotonically increasing function approaching an asymptote of $6U^2 - 3 + \cdots$ for large U^2. Therefore, the poloidal ion and impurity flows are enhanced by the pedestal electric field for normal T_i profiles. The function L_p increases to a maximum of 1.46 at $U^2 = 0.83$ before going to zero exponentially. This increase in L_p enhances the ion heat transport for normal T_i profiles for moderate U^2.

4.2. Banana regime

The banana regime calculation is somewhat more involved because of the need to deal with the collision operator in detail while distinguishing carefully between transit (performed at fixed ψ_*) and flux surface (performed at fixed ψ) averages. The Rosenbluth potential form of the ion–ion collision operator for collisions with a background Maxwellian is used, with the flux function k being determined by the need to conserve momentum in ion–ion collisions so that ion particle transport does not occur. The convenient variables to employ first are the ψ, ϑ, W and Λ variables. Only scattering normal to the trapped-passing boundary $\kappa^2 = 1$ need be retained because we assume $\sqrt{\varepsilon} \ll 1$ and neglect order ε corrections [18]. As a result, we need only evaluate the modified pitch angle scattering operator

$$C\{f\} = \frac{1}{j}\frac{\partial}{\partial \Lambda}\left[j f_{\rm M} \nabla_{\rm v}\Lambda \cdot \overset{\leftrightarrow}{\boldsymbol{Q}} \cdot \nabla_{\rm v}\Lambda \frac{\partial}{\partial \Lambda}\left(\frac{f}{f_{\rm M}}\right)\right], \tag{28}$$

where $j = 1/\nabla_{\rm v} W \times \nabla_{\rm v}\Lambda \cdot \nabla_{\rm v}\varphi$ is the Jacobian and $\overset{\leftrightarrow}{\boldsymbol{Q}} = (\nu_\perp/4)(v^2 \overset{\leftrightarrow}{\boldsymbol{I}} - \boldsymbol{vv}) + (\nu_\parallel/2)\boldsymbol{vv}$, with $\nu_\perp/\nu_{\rm i} = 3(2\pi)^{1/2}[\mathrm{erf}(x) - \Psi(x)]/(2x^3)$, $\nu_\parallel/\nu_{\rm i} = 3(2\pi)^{1/2}\Psi(x)/(2x^3)$, $x = v/v_{\rm i}$, $\mathrm{erf}(x)$ the error function, $\Psi(x) = [\mathrm{erf}(x) - x\,\mathrm{erf}'(x)]/(2x^2)$, $\nu_{\rm i}$ the Braginski ion–ion collision frequency, and the gyrophase φ giving $\nabla_{\rm v}\varphi = v_\perp^2 \boldsymbol{n} \times \boldsymbol{v}$. To evaluate $\nabla_{\rm v} W$ and $\nabla_{\rm v}\Lambda$ we use (8), ΛW from (9) and (10), and $\nabla_{\rm v}\psi_* = I\boldsymbol{n}/\Omega_{\rm i}$ from (1) to obtain

$$(1 - \Lambda B/B_0)\nabla_{\rm v} W = (B/B_0) W \nabla_{\rm v}\Lambda + (v_\parallel + u)\boldsymbol{n} \tag{29}$$

and

$$W\nabla_{\rm v}\Lambda + \Lambda\nabla_{\rm v} W = [(S_0 B_0/B) - 2(B_0/B)(S_0 - 1)]\boldsymbol{v}_\perp + 2(S_0 - 1)\boldsymbol{v}. \tag{30}$$

Upon neglecting order ε corrections the preceding give

$$j = BW/S_0 B_0 (v_\parallel + u) \tag{31}$$

and

$$\begin{aligned}\nabla_{\rm v}\Lambda &= -(\Lambda/W)(v_\parallel + u)\boldsymbol{n} + (1 - \Lambda B/B_0)[S_0\boldsymbol{v}_\perp + 2(S_0 - 1)v_\parallel \boldsymbol{n} + \cdots] \\ &\approx -(\Lambda/W)(v_\parallel + u)\boldsymbol{n}.\end{aligned} \tag{32}$$

The last form of $\nabla_{\rm v}\Lambda$ is all that is needed to evaluate (28), which becomes

$$C_\ell\{H\} = \frac{(v_\parallel + u)}{B}\frac{\partial}{\partial\Lambda}\left[B\Lambda W^{-2}(v_\parallel + u)\boldsymbol{n} \cdot \overset{\leftrightarrow}{\boldsymbol{Q}} \cdot \boldsymbol{n} f_{\rm M}\frac{\partial}{\partial \Lambda}\left(\frac{H}{f_{\rm M}}\right)\right], \tag{33}$$

where to reduce to the usual $u = 0$ result we use $\Lambda \approx 1$, since (33) is only employed for the trapped and barely passing. Equation (33) is easily written in terms of the ψ_*, θ, W and Λ variables by using (5), (6) and (8)–(10) so the transit averages can be performed:

$$C_\ell\{H\} = \frac{(Sv_\parallel + u_*)}{B}\frac{\partial}{\partial\Lambda}\left[B\Lambda W^{-2}(Sv_\parallel + u_*)\boldsymbol{n} \cdot \overset{\leftrightarrow}{\boldsymbol{Q}} \cdot \boldsymbol{n} f_{\rm M}\frac{\partial}{\partial \Lambda}\left(\frac{H}{f_{\rm M}}\right)\right], \tag{34}$$

where $\rho_{\rm pi}/L_{\rm T} \sim B_{\rm p}/B \ll \varepsilon^{1/2}$ is assumed to neglect corrections from $\partial\psi_*/\partial\Lambda$. Note that combining (29) and (32) gives $\nabla_v W \approx (v_\parallel + u)\vec{n}$ and $W\nabla_{\rm v} W \cdot \nabla_{\rm v}\Lambda \approx -\Lambda(v_\parallel + u)^2$. As a result, the neglected velocity space derivatives from the full ion–ion collision operator are expected to be small as Λ derivatives acting on H are larger than W derivatives for $\varepsilon \ll 1$.

To complete the specification of the collision operator we need to evaluate the coefficient $\boldsymbol{n} \cdot \overset{\leftrightarrow}{\boldsymbol{Q}} \cdot \boldsymbol{n}$ for the trapped and barely passing ($v_\parallel + u = Sv_\parallel + u_* \approx 0$) in both sets of variables. Using $v^2 = v_\parallel^2 + 2\mu B$ and the lowest order result $v_\parallel \approx -u \approx -u_*/S$, (6) and (9) then give $E_* \approx \mu B + u_*^2/2S \approx \mu B + Su^2/2$ and $W \approx S_0\mu B_0 + u_{*0}^2/S_0 \approx S_0\mu B_0 + S_0 u^2$, since here we can neglect $\varepsilon^{1/2}$ order corrections. Using these results gives $v^2 \approx 2W/S - u^2 \approx 2W/S_0 - u_{*0}^2/S_0^2$, which enters in $\nu_\perp$ and $\nu_{||}$ and allows us to write $\boldsymbol{n}\cdot\overset{\leftrightarrow}{\boldsymbol{Q}}\cdot\boldsymbol{n}$ in both sets of variables:

$$\boldsymbol{n}\cdot\overset{\leftrightarrow}{\boldsymbol{Q}}\cdot\boldsymbol{n} \approx W\nu_\perp/2S + (\nu_\| - \nu_\perp)u^2/2 \approx W\nu_\perp/2S_0 + (\nu_\| - \nu_\perp)u_{*0}^2/2S_0^2. \tag{35}$$

Recalling (20) and using the lowest order banana regime result $\partial h/\partial\vartheta = 0$, we see that the transit average of the next order version of (20) with (34) inserted for C for the passing ions allows the lowest order flux function $\partial h/\partial\Lambda$ to be determined from

$$\left\langle \frac{BC_\ell\{H\}}{Sv_\| + u_*} \right\rangle = 0. \tag{36}$$

As usual, $h = 0$ for the trapped particles since the transit average is over a full bounce. Note that the transit averages can essentially be performed in the usual manner [6–8, 18]. Once $\partial h/\partial\Lambda$ is determined from (36) for the passing in the ψ_*, θ, W and Λ variables it is easily rewritten in the ψ, θ, W and Λ variables, and the desired moments can be formed in the conventional manner. The flux function k is determined by the need to conserve momentum in ion–ion collisions and is found to be

$$\frac{3k}{7} \equiv A_{\rm b}(U^2) = \frac{6}{7}\left[\frac{5}{2} + U^2 - \frac{\int_0^\infty {\rm d}y{\rm e}^{-y}(y+2U^2)^{3/2}(y\nu_\perp + 2U^2\nu_\|)}{\int_0^\infty {\rm d}y{\rm e}^{-y}(y+2U^2)^{1/2}(y\nu_\perp + 2U^2\nu_\|)}\right]. \tag{37}$$

The details of this evaluation as well as those for the parallel ion flow velocity, radial ion heat flux and bootstrap current are found in [1, 2] and lead to the following banana regime expressions (denoted by an index b):

$$n_{\rm i}V_{\|{\rm i}}^{\rm b} = -\frac{Ip_{\rm i}}{M\Omega_{\rm i}}\left[\frac{1}{p_{\rm i}}\frac{\partial p_{\rm i}}{\partial\psi} + \frac{Ze}{T_{\rm i}}\frac{\partial\Phi}{\partial\psi} - \frac{7A_{\rm b}(U^2)B^2}{6\langle B^2\rangle T_{\rm i}}\frac{\partial T_{\rm i}}{\partial\psi}\right], \tag{38}$$

$$\langle \boldsymbol{q}_{\rm i}^{\rm b}\cdot\nabla\psi\rangle = -1.35\frac{\sqrt{\varepsilon}\nu_{\rm i}I^2p_{\rm i}}{M\Omega_{\rm i}^2}\frac{\partial T_{\rm i}}{\partial\psi}\frac{L_{\rm b}(U^2)}{\sqrt{S}} \tag{39}$$

and

$$J_{\rm bs}^{\rm b} = -1.46\sqrt{\varepsilon}\frac{cIB}{\langle B^2\rangle}\frac{(Z^2+2.21Z+0.75)}{Z(Z+\sqrt{2})} \times\left[\frac{\partial p}{\partial\psi} - \frac{(2.07Z+0.88)n_{\rm e}}{(Z^2+2.21Z+0.75)}\frac{\partial T_{\rm e}}{\partial\psi} - \frac{7A_{\rm b}(U^2)n_{\rm e}}{6Z}\frac{\partial T_{\rm i}}{\partial\psi}\right], \tag{40}$$

where

$$L_{\rm b}(U^2) = 1.53{\rm e}^{-U^2}\int_0^\infty {\rm d}y{\rm e}^{-y}(y+2U^2)^{3/2}(y+U^2)^{-3/2}(y+U^2-5/2+7J_{\rm b}/6) \times[y\,{\rm erf}(\sqrt{y+U^2}) + (2U^2-y)\Psi(\sqrt{y+U^2})]. \tag{41}$$

The poloidal flow of Pfirsch–Schluter trace impurities and banana regime ions is now given by

$$V_{\rm z}^{\rm b}\big|_{\rm pol} = -\frac{cIB_{\rm p}T_{\rm i}}{Ze\langle B^2\rangle}\left[\frac{1}{p_{\rm i}}\frac{\partial p_{\rm i}}{\partial\psi} - \frac{ZT_{\rm z}}{Z_{\rm z}T_{\rm i}p_{\rm z}}\frac{\partial p_{\rm z}}{\partial\psi} - \frac{7A_{\rm p}(U^2)}{6T_{\rm i}}\frac{\partial T_{\rm i}}{\partial\psi}\right]. \tag{42}$$

As in the plateau regime, the normalized factors $A_{\rm b}$ and $L_{\rm b}$ are the modifications of the usual banana regime results [8] caused by finite radial electric field effects. The bootstrap current is evaluated by inserting the $A_{\rm b}$ modification in the expressions given by Helander and Sigmar [8]. The function $A_{\rm b}$ decreases monotonically, changing sign at $U^2 = 1.44$. As a result, the poloidal ion flow can differ greatly from the usual banana regime result. The predicted change in the flow of the background ions also impacts the impurity flow as given by (42) and is indeed observed on Alcator C-Mod [13, 19]. The function L_b starts out rather flat at small U^2 before going to zero exponentially. Consequently, the radial ion heat flux is reduced by the finite electric field that acts to decrease the number of trapped and barely passing ions.

5. Zonal flow residual

Analysis of the zonal flow residual is a way of considering the response to a turbulence generated, axisymmetric small amplitude, zonal flow potential fluctuation. This perturbation is assumed to have rapid radial spatial variation, but no poloidal variation: $\Phi(\psi, t) = \tilde{\Phi}(t)\exp[\mathrm{i}G(\psi)]$. The turbulence is assumed to generate the change in the potential in a time that is long compared with an ion gyro-period, but short compared with an ion bounce time, whilst not perturbing the density. The standard definition for the ion zonal flow residual is the ratio of the long time asymptotic value of the perturbed potential to the initial perturbation. It can be obtained by solving (18) in the collisionless limit in the absence of any neoclassical drive ($f_* = f_M$ is employed since the zonal flow and neoclassical problems are additive):

$$\partial h/\partial t + (v_{||} + u)\boldsymbol{n}\cdot\nabla\vartheta\,\partial h/\partial\vartheta = (Ze/T)(\partial\tilde{\Phi}/\partial t) f_M \exp(\mathrm{i}G). \tag{43}$$

We solve (43) by assuming to the lowest order $\partial h/\partial\vartheta = 0$ and then annihilating the streaming term to next order by employing the transit average along the actual trajectory over a full poloidal transit. Using the definition

$$\bar{A} = \frac{\oint \mathrm{d}\vartheta\, A/(v_\| + u)\vec{n}\cdot\nabla\vartheta}{\oint \mathrm{d}\vartheta/(v_\| + u)\vec{n}\cdot\nabla\vartheta} = \frac{\oint \mathrm{d}\vartheta\, A/(Sv_\| + u_*)\vec{n}\cdot\nabla\vartheta}{\oint \mathrm{d}\vartheta/(Sv_\| + u_*)\vec{n}\cdot\nabla\vartheta}, \tag{44}$$

and periodicity in ϑ, we obtain the solution

$$h = (Ze/T_i)\tilde{\Phi} f_M \overline{\exp(\mathrm{i}G)}, \tag{45}$$

where the transit average of the eikonal G must be performed holding ψ_*, W and Λ fixed.

Assuming $Ze\tilde{\Phi}/T_i \ll 1$ and forming the flux surface averaged perturbed ion density $\tilde{n}_i = \langle\int \mathrm{d}^3v[h\,\exp(-\mathrm{i}G) - (Ze\tilde{\Phi} f_M/T_i)]\rangle$ gives

$$\tilde{n}_i = (Ze\tilde{\Phi}/T_i)\left\langle\int \mathrm{d}^3v f_M[\overline{\exp(\mathrm{i}G)}\exp(-\mathrm{i}G) - 1]\right\rangle. \tag{46}$$

If the gyroradius, as well as drift, departure polarization effects are retained then the preceding expression becomes

$$\tilde{n}_i = (Ze\tilde{\Phi}/T_i)\left\langle\int \mathrm{d}^3v f_M[\overline{J_0\,\exp(\mathrm{i}G)}J_0\,\exp(-\mathrm{i}G) - 1]\right\rangle, \tag{47}$$

where the Bessel function $J_0 = J_0(k_r v_\perp/\Omega_i)$ has $k_r = |\nabla G| = RB_p G'$. For a turbulent change to $\tilde{\Phi}(t = 0)$ at $t = 0$ in a time much less the transit time (so the ions have gyrated many times, but not yet drifted significantly), the initial density $\tilde{n}_i$ of (47) is simply

$$\tilde{n}_i(t = 0) = (Ze\tilde{\Phi}(t = 0)/T_i)\left\langle\int \mathrm{d}^3v f_M[\overline{J_0}J_0 - 1]\right\rangle. \tag{48}$$

If we assume this change in $\tilde{\Phi}$ leaves the ion density unchanged and wait for $\tilde{\Phi}$ to settle to its time asymptotic value $\tilde{\Phi}(t \to \infty)$, while keeping $\tilde{n}_i(t \to \infty) = \tilde{n}_i(t = 0)$, then we can form the ratio of (47) and (48) to obtain

$$\frac{\tilde{\Phi}(t \to \infty)}{\tilde{\Phi}(t = 0)} = \frac{\left\langle \int d^3v f_M[\overline{J_0} J_0 - 1] \right\rangle}{\left\langle \int d^3v f_M[\overline{J_0 \exp(iG)} J_0 \exp(-iG) - 1] \right\rangle}. \tag{49}$$

For small k_r and u (49) reduces to the form of Rosenbluth and Hinton [9, 10].

To extend their result to finite u we need to Taylor expand G in a way that the lowest order term depends only on ψ_*, W, Λ and constants, and the next corrections are small in $\sqrt{\varepsilon}$ so that exponentials can be expanded in (49). We start by writing

$$\psi = \psi_* - (I u_*/S\Omega_i) + [I(Sv_\parallel + u_*)/S\Omega_i], \tag{50}$$

so the last term is a $\sqrt{\varepsilon}$ correction for the trapped and barely passing ions—the only ones of interest for our evaluation. Taking account of the poloidal variation of B in u and S as well as Ω_i and using the ion orbit results of section 2 to retain order ε terms, we can write

$$\psi = \Psi_* + [I(Sv_\parallel + u_*)/S\Omega_i] + [4\varepsilon I u_{*0}/S_0^2\Omega_{0i}\kappa^2][1 - \kappa^2 \sin^2(\vartheta/2)] + \cdots, \tag{51}$$

where $\Psi_* \equiv \psi_* - (I u_{*0}/S_0\Omega_{0i})[1 - 4\varepsilon/S_0^2\kappa^2]$. We then Taylor expand G about Ψ_* to obtain

$$G(\psi) = G_* + P + \cdots = G_* + Q - L + \cdots, \tag{52}$$

where $G_* = G(\Psi_*)$, $G'_* = G'(\Psi_*)$, $P = Q - L$, $Q = [I(Sv_\parallel + u_*)/S\Omega_i]G'_*$ and $L = [4\varepsilon I u_{*0}/S_0^2\Omega_{0i}\kappa^2][1 - \kappa^2 \sin^2(\vartheta/2)]G'_*$. We neglect G'' and smaller terms when performing the Taylor expansion because the eikonal has the usual property that $|\nabla\nabla G| \sim k_r/a$. Expanding (49) for small k_r then gives

$$\frac{\tilde{\Phi}(t \to \infty)}{\tilde{\Phi}(t = 0)} = \frac{1}{1 + \Re}, \tag{53}$$

with

$$\Re = 2n_i^{-1}\langle k_r^2\rho_i^2\rangle^{-1} \left\langle \int d^3v f_M[i(P - \bar{P}) + (P^2 - 2P\bar{P} + \overline{P^2})/2] \right\rangle \tag{54}$$

to the requisite order. Noting that $L \sim \sqrt{\varepsilon} Q$, then only linear terms in L need be retained in (54) along with linear and quadratic terms in Q. The evaluation then proceeds as in Landreman and Catto [5], where the full details are presented, to find

$$\Re = \Re_{RH}\left[\frac{\Gamma(U^2)}{\sqrt{S}} + i\frac{\Lambda(U, S)}{\langle k_r\rho_{pi}\rangle}\right], \tag{55}$$

where $\Re_{RH} = 1.6q^2/\sqrt{\varepsilon}$ is the Rosenbluth and Hinton result,

$$\Gamma(U^2) = (4/3\sqrt{\pi})\exp(-U^2)\int_0^\infty dy(y + 2U^2)^{3/2}\exp(-y) \tag{56}$$

and

$$\Lambda(U, S) = 2S^{-1/2}U\left[S\Gamma(U^2) + 4\pi^{-1/2}(1 - S)\exp(-U^2)\int_0^\infty dy(y + 2U^2)^{1/2}\exp(-y)\right]. \tag{57}$$

The exponential decay of Γ and Λ for large U is due to the shift of the trapped region to the tail of the distribution function, orbit squeezing effects only enter algebraically in Λ, and the imaginary term is a spatial phase shift in $\tilde{\Phi}$ introduced by u.

6. Discussion

We have presented a streamlined evaluation of the ion orbits and a generalized kinetic treatment that allows us to recover all the pedestal results obtained to date for neoclassical ion flow and heat flux, and the bootstrap current in the banana [1, 2] and plateau [3] regimes, and for the zonal flow residual in the collisionless limit [4, 5]. The techniques we employ clearly distinguish between trajectory and flux surface averages and unify recent evaluations of pedestal phenomena. These modifications include the banana regime decrease and plateau regime alterations in the ion heat diffusivity, the reduction and possible reversal of the poloidal flow in the banana regime and its augmentation in the plateau regime, the increase in the bootstrap current, and the enhancement of the residual zonal flow regulation of turbulence.

There has been other recent neoclassical work considering the $\boldsymbol{E} \times \boldsymbol{B}$ flow velocity to be on the order of the ion thermal speed times B_{p}/B [20]—the same ordering we use here for the radial electric field. However, unlike the ordering we employ, their orderings assume that diamagnetic drifts are much smaller than the ion thermal speed, since the poloidal ion gyroradius is assumed small compared with the radial scale lengths associated with density, temperature and potential variation. Importantly, this absence of poloidal gyroradius scale lengths means the finite orbit effects treated herein cannot be recovered, and the $\boldsymbol{E} \times \boldsymbol{B}$ and ion diamagnetic drifts do not cancel to lowest order. Thus, the parallel ion flow remains sonic rather than becoming subsonic, so centrifugal effects enter. Indeed, the results are more appropriate to a sonic core [21] and, although obtained by a different procedure, similar to earlier large flow banana regime treatments of neoclassical theory [22–24].

Earlier [25, 26] and recent [27] H mode pedestal work found pressure and ion temperature gradient terms in the ion flow that depend on the orbit squeezing factor S, but this form is inconsistent with both (i) the isothermal ion limit for which the ions must be an exact toroidally rotating Maxwellian to make the Vlasov and ion–ion collision operators vanish [28] (so the flow cannot depend on S), and (ii) the results presented here which find that orbit squeezing does not enter the lowest order ion flow. The error can be traced back to the use of an approximate solution for the lowest order ion distribution function, which does not depend on just the constants of the motion [25]. Even more recent work considers another 'quasi-equilibrium' to determine modifications to the parallel flow rather than 'attempt an ambitious problem of analytically solving a drift-kinetic equation' [29] to treat finite orbit effects.

The earliest orbit squeezing evaluation of ion heat transport [30] differs from ours because finite radial electric field effects are incompletely treated by expanding about $\Phi' = 0$ (or $u = 0$); an expansion not appropriate in an H mode pedestal. Moreover, the orbit squeezing dependence of the ion diffusivity is found to be proportional to $S^{-3/2}$ (rather than the $S^{-1/2}$ as found here), due to momentum conservation in ion–ion collisions being imposed by holding the canonical angular momentum ψ_* fixed rather than the magnetic flux ψ (this sublety is addressed in detail in appendix B of [1]). The criticism by [26] of the finite orbit treatment in [30] is avoided in our work by using canonical angular momentum as the radial variable rather than the magnetic flux.

Recent comparisons of our orbit squeezing independent plateau (27) and banana (42) regime expressions for the poloidal impurity flow in the pedestal of Alcator C-Mod have demonstrated that finite electric field effects are modest in the plateau regime, but noticeably improve the agreement between theory and experiment in a banana regime pedestal of poloidal ion gyroradius width [19]. Consequently, our finite electric field modified banana regime expression for the bootstrap current (40), which also depends on the ion flow, must be viewed as an improvement over conventional expressions in an H mode pedestal.

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