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Corrigendum

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In the calculation of the perturbed ion distribution, a contribution to the source term on the right side of equation (19) was missed because $\mathbf{b} \cdot \nabla(v^2/2)$ evaluated at fixed $\mathcal{E} = S \frac{v_{\parallel}^2}{2} + \mu B + v_{\parallel} u_*$ was inadvertently assumed to vanish. Retaining the terms from

$$\begin{aligned} (Sv_{\parallel} + u_*)\mathbf{b} \cdot \nabla(v^2/2) &= (Sv_{\parallel} + u_*)\mathbf{b} \cdot \nabla[(1-S)v_{\parallel}^2/2 - v_{\parallel}u_*] \\ &= [u_* + (S-1)v_{\parallel}](\mu B + v_{\parallel}^2)\mathbf{b} \cdot \nabla \ln B, \end{aligned}$$

then equation (20) should read

$$\begin{aligned} Q_i &= \epsilon v_i f_{Mi} \left\{ \frac{2x_{\parallel}^2 + x_{\perp}^2}{2\Omega_i} I \left[x_{\perp}^2 - 2x_{\parallel} \frac{u_*}{v_i} + x_{\parallel}^2(3-2S) - \frac{5}{2} \right] \frac{\partial \ln T_i}{\partial \Psi} \right. \\ &\quad \left. + \frac{MBk}{2T_i} \left[x_{\perp}^2 - (4S-2)x_{\parallel}^2 - 4x_{\parallel} \frac{u_*}{v_i} \right] \right\}. \end{aligned} \quad (101)$$

The expression for the ion particle flux is affected by the correction. Equation (25) should be replaced by

$$\begin{aligned} \langle \Gamma_i \cdot \nabla \Psi \rangle &\approx -\sqrt{\frac{\pi}{2}} \frac{I^2 \epsilon^2 n_i}{\Omega_i^2 q R_0} \left(\frac{T_i}{M} \right)^{3/2} e^{-U^2} \left\{ \left(\frac{1}{2} + 2U^2 + 3U^4 + 6U^6 \right) \frac{\partial \ln T_i}{\partial \Psi} \right. \\ &\quad \left. + [1 + 2(U^2 + U^4)] \frac{Zek\langle B^2 \rangle}{IT_i c} \right\}. \end{aligned} \quad (102)$$

Requiring this new expression to vanish determines k in equation (26). The corrected J that results replaces equation (27) with

$$J(U^2) = \frac{1 + 4U^2 + 6U^4 + 12U^6}{1 + 2(U^2 + U^4)}. \quad (103)$$

The parallel bulk ion flow, impurity poloidal flow, bootstrap current, electron particle flux, and electron heat flux are still given by equations (31), (32), (50), (A4), and (A5) respectively, but with $J(U^2)$ given by (103) instead of (27). Evaluating the radial ion heat flux by substituting equation (101) together with with equation (103) into equation (28) leads to the previously published result given by equations (29) and (30).

The new $J(U^2)$ is a monotonically increasing function approaching an asymptote of $6U^2 - 3 + \mathcal{O}(U^{-2})$ as U^2 goes to infinity. Figure 1 must be replaced by the new figure 4 given here. The electron flux can be neglected in the ambipolarity constraint for k now only if $(1/2 + 2U^2 + 3U^4 + 6U^6) \exp(-U^2) \gg \sqrt{m/M}$. The maximum value of $|U|$ thus allowed is approximately 3.5, unchanged from the value originally reported.

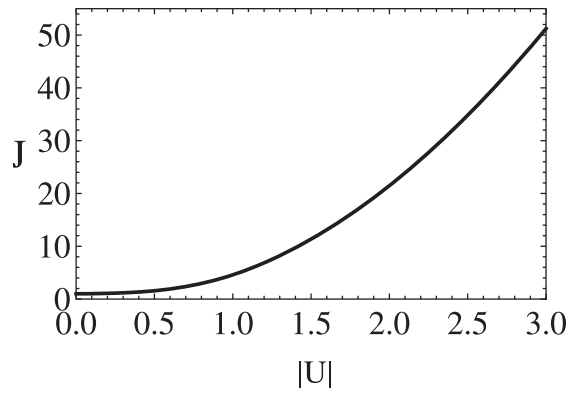


Figure 4. The $J(|U|)$ factor multiplying the ion temperature gradient term in the ion flow.

The corrected $J(U^2)$ function given in equation (103) increases significantly even below $|U| \sim 1.5$ compared to our previous result. Consequently, the statement that ‘extremely precise measurements are required for quantitative comparisons in the plateau regime’ might be relaxed.

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Neoclassical plateau regime transport in a tokamak pedestal

Istvan Pusztai¹ and Peter J Catto²

¹ Nuclear Engineering, Applied Physics, Chalmers University of Technology and Euratom-VR Association, SE-41296 Göteborg, Sweden

² Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

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Abstract

In tokamak pedestals with subsonic flows the radial scale of plasma profiles can be comparable to the ion poloidal Larmor radius, thereby making the radial electrostatic field so strong that the $E \times B$ drift has to be retained in the ion kinetic equation in the same order as the parallel streaming. The modifications of neoclassical plateau regime transport—such as the ion heat flux and the poloidal ion and impurity flows—are evaluated in the presence of a strong radial electric field. The altered poloidal ion flow can lead to a significant increase in the bootstrap current in the pedestal where the spatial profile variation is strong because of the enhanced coefficient of the ion temperature gradient term near the electric field minimum. Unlike the banana regime, orbit squeezing does not affect the plateau regime results.

1. Introduction

High confinement mode (H-mode) operation, characterized by the presence of an edge transport barrier called a pedestal [1], or other improved confinement scenarios are currently considered to be favorable in future fusion reactors [2]. One of the most important open questions of fusion plasma theory is the physics of pedestal and internal transport barriers.

Although it is commonly assumed that turbulent transport dominates in tokamak plasmas, some neoclassical [3] effects are also significant, particularly in the pedestal, where the turbulence is reduced. The ion heat flux can be comparable to the neoclassical predictions in the transport barrier regions [4, 5]. Even though transport barrier formation cannot be explained by long wavelength neoclassical (non-zonal) sheared flow within the flux surface, these flows are predicted [6, 7] and experimentally found to play an important role in the suppression of turbulence in transport barriers [8–11]. Even in turbulence dominated plasmas the relation between the parallel ion flow and the global radial electric field is expected to be neoclassical with a negligible turbulent contribution [12, 13]. Experimental tests of predictions

for bulk ion and impurity flows based on numerical and analytical neoclassical models have been reported in [14–17]. Furthermore, bootstrap current (sections 11.4 and 12.7 in [3]) plays a crucial role in reactor relevant operation of fusion devices, by reducing the need for inductive current drive [18].

Experimental results show that in tokamak pedestals with subsonic ion flows the radial scale of plasma profiles can be as small as the ion poloidal Larmor radius [17] of the main ion species. Accordingly, the radial electrostatic field is so strong ($\sim 100 \text{ kV m}^{-1}$, up to 300 kV m^{-1} on Alcator C-Mod [14]) that the contribution of the $\mathbf{E} \times \mathbf{B}$ drift to the poloidal ion motion is comparable to that of the parallel streaming. Therefore, these two components of the poloidal motion have to be retained in the ion kinetic equation in the same order. The conventional way of solving the drift kinetic equation by expressing the cross-field magnetic drifts in terms of a parallel gradient has to be modified, since this method cannot be applied to the $\mathbf{E} \times \mathbf{B}$ drifts.

The kinetic description of plasmas has recently been extended for pedestals with strong electric fields and short scale lengths by making use of the fact that the canonical angular momentum is a constant of the motion, and using it as the radial coordinate instead of the poloidal flux [19]. By adopting this approach herein modifications to neoclassical plateau regime transport [20, 21] are evaluated in the pedestal regions. These results are relevant in existing fusion devices such as Alcator C-Mod [14]. The calculation allows for $\mathcal{O}(1)$ values of the normalized electric field $U = v_{\mathbf{E} \times \mathbf{B}} B / (v_i B_{\text{pol}})$, where $v_{\mathbf{E} \times \mathbf{B}}$ is the $\mathbf{E} \times \mathbf{B}$ velocity, v_i is the ion thermal speed and B and B_{pol} are the magnitudes of the total and poloidal magnetic fields, respectively, with $B_{\text{pol}} \ll B$.

We find that the ion heat diffusivity is reduced for large values of U , as the resonance causing plateau regime transport is shifted toward the tail of the distribution, but it is enhanced if $U \approx 1$. Moreover, the poloidal ion and impurity flows are modified in the pedestal. The altered poloidal ion flow is most pronounced in the region of the strongest radial electric field where it modifies the friction of the electrons with the ions and can lead to an increase in the bootstrap current, by enhancing the coefficient of the ion temperature gradient term. Unlike the banana regime [22], orbit squeezing does not affect the plateau regime results.

In the next section, the perturbed ion distribution function is derived and the ion heat flux is calculated together with the parallel ion flow and the poloidal impurity rotation. The bootstrap current is evaluated in section 3. Finally the results are summarized in section 4.

2. Ion transport and parallel ion flow

The magnetic field is represented as $\mathbf{B} = I \nabla \zeta + \nabla \zeta \times \nabla \Psi$, where ζ is the toroidal angle, $2\pi \Psi$ is the poloidal flux and $I(\Psi)$ is defined by $\mathbf{B}_t = I \nabla \zeta$, where \mathbf{B}_t is the toroidal component of the magnetic field and $|\nabla \zeta| = 1/R$. The radial electric field E_r is assumed to be of the magnitude $\sim B_p v_i / c$, so the $\mathbf{E} \times \mathbf{B}$ drift is kept in the ion kinetic equation in the same order as the parallel streaming since we assume $B_p \ll B$. Throughout the calculation we assume a quadratic electric potential well

$$\Phi(\Psi) = \Phi(\Psi_*) + (\Psi - \Psi_*) \Phi'(\Psi_*) + \frac{1}{2} (\Psi - \Psi_*)^2 \Phi''(\Psi_*), \quad (1)$$

where the preceding convenient representation makes use of

$$\Psi_* = \Psi - \frac{Mc}{Ze} R v \cdot \hat{\zeta} = \Psi - \frac{I v_{\parallel}}{\Omega} + \frac{\mathbf{v} \times \mathbf{b} \cdot \nabla \Psi}{\Omega}, \quad (2)$$

where M is the mass and Ze is the charge of the particle, R is the major radius, $\Omega = ZeB/Mc$ is the cyclotron frequency and $\mathbf{b} = \mathbf{B}/B$. Within a constant multiplier the canonical angular momentum is Ψ_* , and it is a constant of the motion due to axisymmetry. The ratio of the last two terms on the right side of equation (2) is $B/B_p \gg 1$, so we take $\Psi_* \approx \Psi - I v_{\parallel} / \Omega$.

It is convenient to introduce $u = cI\Phi'/B$, together with $u_* = cI\Phi'_*/B$, where $\Phi'_* = \Phi'(\Psi_*)$. The $\mathbf{E} \times \mathbf{B}$ drift competes with the parallel streaming when they have comparable projections in the poloidal plane—the situation we consider here. The quantity u is sometimes referred to as the $\mathbf{E} \times \mathbf{B}$ drift in the poloidal magnetic field. We assume B is slowly varying with Ψ , so that $B(\Psi_*, \theta) \approx B(\Psi, \theta)$. The orbit squeezing factor $S = 1 + cI^2\Phi''_*/(B\Omega)$ is considered to be constant except for its B dependence. Using the preceding notation the poloidal motion of the particles is given by

$$\dot{\theta} = (v_{\parallel}\mathbf{b} + \mathbf{v}_{E \times B}) \cdot \nabla\theta = (v_{\parallel} + u)\mathbf{b} \cdot \nabla\theta \approx (Sv_{\parallel} + u_*)/(qR), \quad (3)$$

where θ is the poloidal angle and q is the safety factor. To find the final form of equation (3) we use equation (1) to obtain

$$u_*(\Psi_*) = \frac{cI}{B}[\Phi'(\Psi) + (\Psi_* - \Psi)\Phi''(\Psi)] = u(\Psi) - \frac{cI^2v_{\parallel}\Phi''(\Psi)}{B\Omega} = u + (1 - S)v_{\parallel}. \quad (4)$$

We adopt the treatment of [19] and use Ψ_* as radial coordinate, rather than the poloidal flux function Ψ , thereby allowing for the handling of strong gradients in the plasma profiles and the electrostatic potential. A convenient energy variable is then defined as

$$\mathcal{E} = \frac{v^2}{2} + \frac{Ze}{M}[\Phi(\Psi) - \Phi(\Psi_*)] = E - \frac{Ze}{M}\Phi(\Psi_*) = S\frac{v_{\parallel}^2}{2} + \mu B + v_{\parallel}u_*, \quad (5)$$

where $E = v^2/2 + Ze\Phi/M$ is the total energy and $\mu = v_{\perp}^2/2B$ is the magnetic moment. Note that \mathcal{E} is conserved by the Vlasov operator $d_t \equiv \partial_t + \mathbf{v} \cdot \nabla + (\Omega\mathbf{v} \times \mathbf{b} - Ze\nabla\Phi/M) \cdot \nabla_v$ since $d_t\mathcal{E} = 0$, because E and Ψ_* are constants of the motion.

In [19] it is shown that the lowest order solution of the electrostatic gyrokinetic equation must be a Maxwellian even in the pedestal where the density and potential can vary on the scale of the poloidal ion Larmor radius. Proceeding to higher order we write the gyro-averaged distribution function as

$$\bar{f} = f_*(\Psi_*, \mathcal{E}) + h(\Psi_*, \mathcal{E}, \mu, \theta, t), \quad (6)$$

where f_* is a distribution function close to a stationary Maxwellian, that is only a function of the constants of the motion \mathcal{E} and Ψ_* . Upon Taylor expanding the slowly varying functions η and T_i as in [19] we obtain

$$\begin{aligned} f_* &= \eta(\Psi_*) \left(\frac{M}{2\pi T_i(\Psi_*)} \right)^{3/2} e^{-\frac{mE}{T_i(\Psi_*)}} \\ &\approx f_{Mi} \left\{ 1 - \frac{Iv_{\parallel}}{\Omega_i} \left[\frac{\partial \ln p_i}{\partial \Psi} + \frac{Ze}{T_i} \frac{\partial \Phi}{\partial \Psi} + \left(\frac{mv^2}{2T_i} - \frac{5}{2} \right) \frac{\partial \ln T_i}{\partial \Psi} \right] + \dots \right\}, \end{aligned} \quad (7)$$

where

$$\eta(\Psi_*) = \eta(\Psi) + (\Psi_* - \Psi) \frac{\partial \eta(\Psi)}{\partial \Psi} + \dots, \quad (8)$$

$$\eta(\Psi) = n_i(\Psi) \exp \left[\frac{Ze\Phi(\Psi)}{T_i(\Psi)} \right], \quad (9)$$

and the stationary Maxwellian on a flux surface is

$$f_{Mi} = n_i(\Psi) \left(\frac{M}{2\pi T_i(\Psi)} \right)^{3/2} e^{-\frac{Mv^2}{2T_i(\Psi)}}. \quad (10)$$

We consider subsonic flows, so that in the pedestal $\partial_{\Psi} \ln n_i \approx -(Ze/T_i)\partial_{\Psi}\Phi$. The governing equation for a time independent perturbed ion distribution, in accordance with [19], is

$$\dot{\theta} \frac{\partial h_{1i}}{\partial \theta} - C_{ii}^1 \left\{ h_{1i} - \frac{Iv_{\parallel}f_{Mi}}{\Omega_i} \left(\frac{Mv^2}{2T_i} - \frac{5}{2} \right) \frac{\partial \ln T_i}{\partial \Psi} \right\} = 0, \quad (11)$$

where C_{ii}^1 is the linearized ion–ion collision operator, which is momentum conserving and the θ derivative is taken keeping \mathcal{E} , Ψ_* and μ fixed. The kinetic equation (11) can be rewritten as

$$(Sv_{\parallel} + u_*)\mathbf{b} \cdot \nabla \left[H_i + \frac{Iv_{\parallel}f_{Mi}}{\Omega_i} \left(\frac{Mv^2}{2T_i} - \frac{5}{2} \right) \frac{\partial \ln T_i}{\partial \Psi} \right] - C_{ii}^1\{H_i\} = 0, \quad (12)$$

where we have introduced

$$H_i = h_{li} - \frac{Iv_{\parallel}f_{Mi}}{\Omega_i} \left(\frac{Mv^2}{2T_i} - \frac{5}{2} \right) \frac{\partial \ln T_i}{\partial \Psi}. \quad (13)$$

In the plateau regime, the form of the collision operator cannot affect the transport when the kinetic equation is written in the form of (12). Therefore, we can use a simple Krook operator to model the collisions. However, the replacement $C_{ii}^1\{H_i\} \rightarrow -\nu H_i$ destroys the momentum conserving property of the operator. This defect is remedied by adding a homogeneous solution to H_i and then determining its free coefficient by making use of the fact that $C_{ii}^1\{v_{\parallel}f_M\} = 0$ [3, 21, 23]. The addition of a term proportional to $v_{\parallel}f_M$ to the perturbed distribution gives a contribution to the parallel ion flow. To make this contribution to the flow divergence free it has to be proportional to the magnetic field strength. Accordingly, we adopt the following replacement:

$$H_i \rightarrow H_i + \frac{MBkv_{\parallel}f_{Mi}}{T_i}, \quad (14)$$

where the unknown k is to be determined by requiring that the solution gives no radial particle flux.

After the replacements the kinetic equation becomes

$$(Sv_{\parallel} + u_*)\mathbf{b} \cdot \nabla H_i + \nu H_i = -(Sv_{\parallel} + u_*)\mathbf{b} \cdot \nabla \left\{ \frac{Iv_{\parallel}f_{Mi}}{\Omega_i} \left(\frac{Mv^2}{2T_i} - \frac{5}{2} \right) \frac{\partial \ln T_i}{\partial \Psi} + \frac{MBkv_{\parallel}f_{Mi}}{T_i} \right\}. \quad (15)$$

The spatial derivatives can be performed by recalling the B dependence of u_* and S to find

$$(Sv_{\parallel} + u_*)\mathbf{b} \cdot \nabla|_{\mathcal{E},\mu,\Psi_*} v_{\parallel} = \left[-\mu B + v_{\parallel}u_* + 2(S-1)\frac{v_{\parallel}^2}{2} \right] \mathbf{b} \cdot \nabla \ln B, \quad (16)$$

which then follows from $\mathbf{b} \cdot \nabla|_{\mathcal{E},\mu,\Psi_*} \mathcal{E} = 0$. Employing equation (16) we find the relations

$$(Sv_{\parallel} + u_*)\mathbf{b} \cdot \nabla|_{\mathcal{E},\mu,\Psi_*} \left(\frac{v_{\parallel}}{\Omega_i} \right) = \frac{1}{2\Omega_i} [2v_{\parallel}^2 + v_{\perp}^2] \mathbf{b} \cdot \nabla \ln R, \quad (17)$$

$$(Sv_{\parallel} + u_*)\mathbf{b} \cdot \nabla|_{\mathcal{E},\mu,\Psi_*} (v_{\parallel}B) = \frac{B}{2} [v_{\perp}^2 - (4S-2)v_{\parallel}^2 - 4v_{\parallel}u_*] \mathbf{b} \cdot \nabla \ln R. \quad (18)$$

The plateau regime only exists for large aspect ratio ($\epsilon \ll 1$, where $\epsilon = r/R_0$ with the minor radius r) and requires $\epsilon^{1/2} \ll v_i q R / v_i \ll 1$. As a result, we can approximate $\mathbf{b} \cdot \nabla \ln R$ by $-\epsilon \sin \theta / (qR)$ to obtain

$$\left(Sx_{\parallel} + \frac{u_*}{v_i} \right) \frac{\partial H_i}{\partial \theta} + \frac{vqR}{v_i} H_i = Q_i \sin \theta, \quad (19)$$

where $x = v/v_i = (x_{\perp}^2 + x_{\parallel}^2)^{1/2}$ is the velocity normalized to the ion thermal speed $v_i = (2T_i/M)^{1/2}$ and

$$Q_i = \epsilon v_i f_{Mi} \left\{ \frac{2x_{\parallel}^2 + x_{\perp}^2}{2\Omega_i} I \left(x^2 - \frac{5}{2} \right) \frac{\partial \ln T_i}{\partial \Psi} + \frac{MBk}{2T_i} \left[x_{\perp}^2 - (4S-2)x_{\parallel}^2 - 4x_{\parallel} \frac{u_*}{v_i} \right] \right\}. \quad (20)$$

Due to the simple model for collisions the kinetic equation (19) can easily be solved to find

$$H = Q_i \frac{\frac{vqR}{v_i} \sin \theta - \left(Sx_{\parallel} + \frac{u_*}{v_i} \right) \cos \theta}{\left(Sx_{\parallel} + \frac{u_*}{v_i} \right)^2 + \left(\frac{vqR}{v_i} \right)^2} \approx Q_i \left[\pi \delta \left(Sx_{\parallel} + \frac{u_*}{v_i} \right) \sin \theta - \frac{\cos \theta}{Sx_{\parallel} + \frac{u_*}{v_i}} \right], \quad (21)$$

since most passing ions are nearly collisionless. The trapped and barely passing ions are collisional, thus collisions only enter to resolve the singularity at $Sx_{\parallel} + u_*/v_i = 0$.

The full gyro-averaged perturbed distribution $\bar{f}_{i1} = \langle f_i - f_{Mi} \rangle_{\varphi}$ is given by

$$\bar{f}_{i1} = h_i - \frac{Iv_{\parallel}}{\Omega} \frac{\partial f_{Mi}}{\partial \Psi} = H_i + \frac{MBkv_{\parallel} f_{Mi}}{T_i} - \frac{Iv_{\parallel} f_{Mi}}{\Omega_i} \left(\frac{\partial \ln p_i}{\partial \Psi} + \frac{Ze}{T_i} \frac{\partial \Phi}{\partial \Psi} \right). \quad (22)$$

Note that from all the terms in \bar{f}_{i1} only the $\propto \sin \theta$ part of H_i has a finite contribution to the cross-field transport fluxes, and it does not depend on the radial electric field, in accordance with the requirement of intrinsic ambipolarity.

In order to determine the unknown k , we now make the radial ion particle transport vanish

$$0 = \langle \Gamma_i \cdot \nabla \Psi \rangle = \left\langle \int d^3v \bar{f}_{i1} v_d \cdot \nabla \Psi \right\rangle \approx - \left\langle \frac{I\epsilon}{2\Omega q R} \int d^3v (2v_{\parallel}^2 + v_{\perp}^2) H_i \sin \theta \right\rangle, \quad (23)$$

where v_d is the magnetic drift velocity. The velocity integral is to be performed holding Ψ constant, thus $H_i(\Psi_*)$ needs to be transformed back to flux surfaces. The orbit squeezing factor S is not affected by the transformation, since Φ'' is considered to be constant, while from (4) we see that $u_*(\Psi_*)$ is replaced by $u(\Psi) + (1 - S)v_{\parallel}$. Accordingly,

$$\delta \left(Sx_{\parallel} + \frac{u_*}{v_i} \right) \rightarrow \delta(x_{\parallel} + U) \quad \text{and} \quad x_{\perp}^2 - (4S - 2)x_{\parallel}^2 - 4x_{\parallel} \frac{u_*}{v_i} \rightarrow x_{\perp}^2 - 2x_{\parallel}^2 - 4x_{\parallel}U, \quad (24)$$

where we introduced $U = u/v_i$. Equation (24) shows that the resulting transport is insensitive to the orbit squeezing. Substituting equations (21) and (22) into equation (23) the integrals can be evaluated yielding

$$\begin{aligned} \langle \Gamma_i \cdot \nabla \Psi \rangle \approx & - \sqrt{\frac{\pi}{2}} \frac{I^2 \epsilon^2 n_i}{\Omega_i^2 q R_0} \left(\frac{T_i}{M} \right)^{3/2} \times e^{-U^2} \left\{ \left(\frac{1}{2} - U^4 + 2U^6 \right) \frac{\partial \ln T_i}{\partial \Psi} \right. \\ & \left. + [1 + 2(U^2 + U^4)] \frac{Zek \langle B^2 \rangle}{IT_i c} \right\}, \end{aligned} \quad (25)$$

where n_i is the ion density. The ambipolarity condition requires that

$$k = - \frac{J(U^2)}{2} \frac{\partial \ln T_i}{\partial \Psi} \frac{IT_i c}{Ze \langle B^2 \rangle}, \quad (26)$$

with

$$J(U^2) = \frac{1 - 2U^4 + 4U^6}{1 + 2(U^2 + U^4)}, \quad (27)$$

which is consistent with the usual ($U = 0$) plateau result. As illustrated in figure 1, $J(U)$ has a minimum of ≈ 0.39 at $|U| \approx 0.76$ and $J \rightarrow 2U^2 - 3 + \mathcal{O}(U^{-2})$ as $|U|$ goes to infinity.

The preceding calculation of J is based on the observation that if we artificially set $k = 0$ in equation (25) the resulting ion particle flux would be much higher than the electron particle

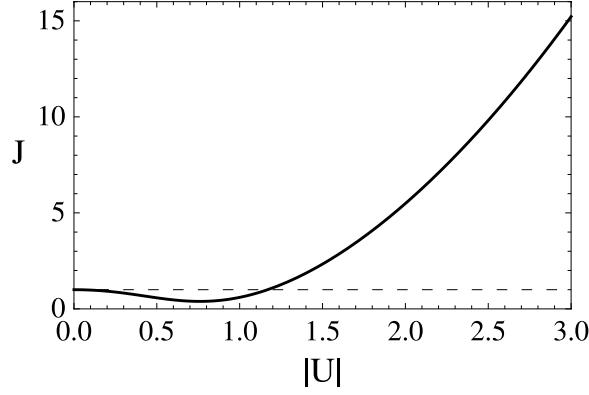


Figure 1. The $J(|U|)$ factor multiplying the ion temperature gradient term in the ion flow.

flux (given in [appendix A](#) for completeness). For $U = 0$ the ion particle flux is larger than the electron particle flux by the square root of the ion to electron mass ratio $\sqrt{m/M}$, so k must be chosen to make equation (25) vanish to lowest order. However, for higher values of U the $\exp(-U^2)$ factor appearing in the expression for the ion particle flux (25) reduces it to the level of neoclassical electron transport. Therefore, our ambipolarity assumption (23) would have to be modified to include the electrons. As this change does not happen until around $U = 3.5$, where $(1/2 - U^4 + 2U^6) \exp(-U^2)$ becomes comparable to $\sqrt{m/M}$, it need not concern us here.

Having calculated the full H_i distribution, we are in a position to evaluate the radial ion heat flux

$$\langle \mathbf{q}_i \cdot \nabla \Psi \rangle = \left\langle \int d^3v \frac{Mv^2}{2} \bar{f}_{i1} \mathbf{v}_d \cdot \nabla \Psi \right\rangle \approx - \left\langle \frac{I\epsilon}{2\Omega q R} \int d^3v \frac{Mv^2}{2} (2v_{\parallel}^2 + v_{\perp}^2) H_i \sin \theta \right\rangle. \quad (28)$$

We find

$$\langle \mathbf{q}_i \cdot \nabla \Psi \rangle \approx -3 \sqrt{\frac{\pi}{2}} \frac{I^2 \epsilon^2 p_i}{\Omega_i^2 q R_0} \left(\frac{T_i}{M} \right)^{3/2} \frac{\partial \ln T_i}{\partial \Psi} L(U^2), \quad (29)$$

with

$$L(U^2) = e^{-U^2} \frac{1 + 4\{U^2 + 2U^4 + [(4U^6 + U^8)/3]\}}{1 + 2(U^2 + U^4)}. \quad (30)$$

The preceding reduces to the standard plateau result [3, 21, 24] in the $U \rightarrow 0$ limit. The function $L(U)$ is plotted in figure 2 to show that plateau ion heat flux is almost 50% higher when the parallel projection of the poloidal component of the ion $\mathbf{E} \times \mathbf{B}$ drift velocity is close to the ion thermal speed [$L(|U| \approx 0.91) \approx 1.46$], but it rapidly drops off for higher values of U , as the resonance causing the plateau transport is shifted toward the tail of the ion distribution.

To calculate the electron transport and the bootstrap current the ion parallel flow needs to be evaluated. Neglecting the small local contribution from H_i we obtain

$$n_i V_{\parallel i} = \int d^3v v_{\parallel} \bar{f}_{i1} \approx - \frac{I p_i}{M \Omega_i} \left[\frac{\partial \ln p_i}{\partial \Psi} + \frac{Ze}{T_i} \frac{\partial \Phi}{\partial \Psi} + \frac{J(U^2)}{2} \frac{B^2}{\langle B^2 \rangle} \frac{\partial \ln T_i}{\partial \Psi} \right]. \quad (31)$$

To relate the poloidal flow of a collisional trace impurity to the poloidal flow of a background ion in the plateau regime we note that the flux surface average of B times their

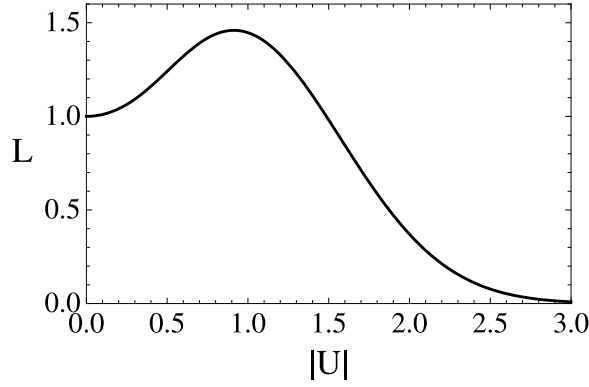


Figure 2. The $L(|U|)$ factor multiplying the plateau ion heat diffusivity.

parallel flows must be related by $\langle BV_{\parallel i} \rangle = \langle BV_{\parallel z} \rangle$ [25, 26]. Using radial pressure balance for the ions and impurities along with the preceding result for $V_{\parallel i}$ gives the impurity poloidal flow to be

$$V_{z,\theta} = \frac{cIT_i B_\theta}{eZ_i \langle B^2 \rangle} \left[\frac{T_z Z_i}{T_i Z_z} \frac{\partial \ln p_z}{\partial \Psi} - \frac{\partial \ln p_i}{\partial \Psi} - \frac{J(U^2)}{2} \frac{\partial \ln T_i}{\partial \Psi} \right]. \quad (32)$$

The preceding expression can be used in C-Mod when the background ions are in the plateau regime [14].

3. Bootstrap current

Even if the radial electric field is high enough to modify the ion transport and flows, i.e. $U = \mathcal{O}(1)$, electron orbits are practically unaffected by the strong radial electric field due to their large thermal speed. However, because electron–ion collisions depend on the ion mean flow, the electron distribution experiences this friction and is thereby indirectly influenced by the presence of the electric field. To evaluate this ion flow effect we next consider the electron problem.

We start with the electron kinetic equation [3]

$$v_{\parallel} \mathbf{b} \cdot \nabla h_{1e} + \frac{e}{T_e} f_{Me} v_{\parallel} E_{\parallel} = C_e^{(1)} \{ \bar{f}_{1e} \}, \quad (33)$$

where the spatial derivatives are performed holding the total energy E and the magnetic moment μ fixed, the parallel induced electric field is denoted by $E_{\parallel} = \mathbf{b} \cdot \nabla (E + \nabla \Phi)$,

$$\bar{f}_{1e} = h_{1e} - \frac{I v_{\parallel}}{\Omega_e} \frac{\partial f_{Me}}{\partial \Psi}, \quad (34)$$

and

$$C_e^{(1)} \{ \bar{f}_{1e} \} = C_{ee}^{(1)} \{ \bar{f}_{1e} \} + \mathcal{L}_{ei} \left\{ \bar{f}_{1e} - \frac{m}{T_e} V_{\parallel i} v_{\parallel} f_{Me} \right\} \quad (35)$$

is the full linearized electron collision operator, with $C_{ee}^{(1)}$ the linearized electron–electron collision operator and \mathcal{L}_{ei} the Lorentz operator (the operators are given in [appendix B](#)). Since $C_{ee}^{(1)} \{ f_{Me} v_{\parallel} \} = 0$, the $V_{\parallel i} v_{\parallel} f_{Me}$ term under \mathcal{L}_{ei} can be added to \bar{f}_{1e} under $C_{ee}^{(1)}$. Introducing f_S ,

the solution of the Spitzer problem

$$C_e^{(1)}\{f_S\} = \frac{e}{T_e} E_1 v_{\parallel} f_{Me}, \quad (36)$$

the kinetic equation (33) can be rewritten as

$$v_{\parallel} \mathbf{b} \cdot \nabla H_e - C_e^{(1)}\{H_e\} = -v_{\parallel} \mathbf{b} \cdot \nabla \left(\frac{I v_{\parallel}}{\Omega_e} \frac{\partial f_{Me}}{\partial \Psi} + \frac{m}{T_e} V_{\parallel i} v_{\parallel} f_{Me} + f_S \right) = A_1 v_{\parallel} \mathbf{b} \cdot \nabla \left(\frac{v_{\parallel}}{\Omega_e} \right) + A_2 v_{\parallel} \mathbf{b} \cdot \nabla (v_{\parallel} B), \quad (37)$$

where

$$H_e = h_{1e} - \frac{I v_{\parallel}}{\Omega_e} \frac{\partial f_{Me}}{\partial \Psi} - \frac{m}{T_e} V_{\parallel i} v_{\parallel} f_{Me} - f_S, \quad (38)$$

$$A_1 = -I f_{Me} \left[\frac{1}{p_e} \frac{\partial p_e}{\partial \Psi} + \frac{1}{Z n_i T_e} \frac{\partial p_i}{\partial \Psi} + \left(\frac{m v^2}{2 T_e} - \frac{5}{2} \right) \frac{1}{T_e} \frac{\partial T_e}{\partial \Psi} \right] \quad (39)$$

and

$$A_2 = -\frac{J(U^2) f_{Mi}}{2 Z T_e} \frac{\partial T_i}{\partial \Psi} \frac{I B}{\Omega_e \langle B^2 \rangle} - \frac{f_S}{v_{\parallel} B}. \quad (40)$$

To get A_2 , we used that the Spitzer function has the form

$$f_S = -\frac{e E_1 v_{\parallel}}{v_e T_e} D(x_e) f_{Me}, \quad (41)$$

and that E_1 is approximately proportional to B . In our notation $\nu_e = 4(2\pi)^{1/2} n_e e^4 \ln \Lambda / (3m^{1/2} T_e^{3/2})$ is the electron–electron collision frequency and D is a dimensionless function of $x_e = v/v_e$, which is parametrically dependent on the ion charge. The function $D(x_e)$ is calculated in [appendix B](#) in terms of generalized Laguerre polynomials, $L_n^{(\lambda)}$, using a variational method to find

$$D(x_e) \approx \sqrt{2} (a_0 L_0^{(3/2)}(x_e^2) + a_1 L_1^{(3/2)}(x_e^2)), \quad (42)$$

where the coefficients are $a_0 = (8 + 13\sqrt{2}Z) / [4Z(2\sqrt{2} + 2Z)]$ and $a_1 = -3 / [2(2 + \sqrt{2}Z)]$.

To evaluate the bootstrap current we begin by noting that the parallel current can be written as

$$j_{\parallel} = \frac{B \langle j_{\parallel} B \rangle}{\langle B^2 \rangle} - \frac{cI}{B} \frac{\partial p}{\partial \Psi} \left(1 - \frac{B^2}{\langle B^2 \rangle} \right), \quad (43)$$

where the second term is the Pfirsch–Schlüter current, and the first term is defined in terms of

$$\langle j_{\parallel} B \rangle = e \left\langle B \int d^3 v v_{\parallel} (Z \bar{f}_{1i} - \bar{f}_{1e}) \right\rangle. \quad (44)$$

Recall that $\langle j_{\parallel} B \rangle$ is the sum of the ohmic current $\langle j_{OH} B \rangle = -e \langle B \int d^3 v v_{\parallel} f_S \rangle = e^2 n_e \langle E_1 B \rangle \sqrt{2} a_0 / (v_e m_e)$ and a current contribution from H_e , the bootstrap current

$$\langle j_{BS} B \rangle = -e \left\langle B \int d^3 v v_{\parallel} H_e \right\rangle. \quad (45)$$

Note that the $\partial_{\Psi} \Phi$ terms from \bar{f}_{1e} and \bar{f}_{1i} cancel in the integrand of equation (44).

The seemingly straightforward procedure for evaluating the bootstrap current would be to solve equation (37) for H_e and substitute it into equation (45). However, H_e is not accurate

enough to give the correct bootstrap current. Instead, it is convenient to use an adjoint method to find $\langle j_{BS} B \rangle$. The adjoint equation that must be solved is given by

$$v_{\parallel} \mathbf{b} \cdot \nabla G + C_e^{(1)}\{G\} = \frac{e}{T_e} E_{\parallel} v_{\parallel} f_{Me}. \quad (46)$$

In terms of G , the bootstrap current can be calculated by the adjoint method as

$$\langle j_{BS} B \rangle = \frac{BT_e}{E_{\parallel}} \left\langle \int d^3v \frac{G}{f_{Me}} \left[A_1 v_{\parallel} \mathbf{b} \cdot \nabla \left(\frac{v_{\parallel}}{\Omega_e} \right) + A_2 v_{\parallel} \mathbf{b} \cdot \nabla (v_{\parallel} B) \right] \right\rangle, \quad (47)$$

This relation is obtained by adding $\langle \int d^3v$ equation (37) G/f_{Me} to $\langle \int d^3v$ equation (46) H_e/f_{Me} and using the self-adjoint property of the linearized collision operator [27, 28]. Writing $G = f_S + g$, the unknown part g gives the only non-zero contribution to equation (47). It is determined by solving

$$v_{\parallel} \mathbf{b} \cdot \nabla g + C_e^{(1)}\{g\} = -\frac{f_S}{B v_{\parallel}} v_{\parallel} \mathbf{b} \cdot \nabla (v_{\parallel} B). \quad (48)$$

Letting $C_e^{(1)}\{g\} \rightarrow -\nu g$, we find the plateau solution

$$g \approx \epsilon \frac{e E_{\parallel} D(x_e)}{\nu_e T_e} f_{Me} \frac{1}{2v} (v^2 - 3v_{\parallel}^2) \left[\pi \delta \left(\frac{v_{\parallel}}{v} \right) \sin \theta + \frac{v \cos \theta}{v_{\parallel}} \right], \quad (49)$$

Substituting the full G into equation (47), evaluating the velocity integrals and performing the flux surface average, we obtain

$$\langle j_{BS} B \rangle = -\sqrt{\frac{\pi}{2}} \frac{\epsilon^2 c I p_e \nu_e}{\nu_e q R_0} \frac{\sqrt{2} + 4Z}{Z(2 + \sqrt{2}Z)} \left\{ \frac{1}{p_e} \frac{\partial p}{\partial \Psi} + \frac{\sqrt{2} + 13Z}{2(\sqrt{2} + 4Z)} \frac{1}{T_e} \frac{\partial T_e}{\partial \Psi} + \frac{J(U^2)}{2Z T_e} \frac{\partial T_i}{\partial \Psi} \right\}, \quad (50)$$

where $p = p_e + p_i$ and we ignore $\epsilon^2 \nu_e / (q R \nu_e) \ll 1$ corrections to the Spitzer current. In calculating the bootstrap current we have kept only two terms in the Laguerre polynomial expansion. It is shown in appendix B that keeping more terms or using a different expansion—as in [29, 30]—would not give significant improvement to the preceding result.

4. Conclusions

In subsonic tokamak pedestals having $B_{pol} \ll B$, due to the strong ($\sim 100 \text{ kV m}^{-1}$) radial electric field, the contribution of the $\mathbf{E} \times \mathbf{B}$ drift to the poloidal ion motion can be comparable to that of the parallel streaming. Thus, these contributions have to be kept in the ion kinetic equation in the same order. In the preceding sections the modifications of the neoclassical plateau regime transport are evaluated, and expressed in terms of the normalized electric field $U = v_{E \times B} B / (v_i B_{pol})$ which is allowed to be order unity. Orbit squeezing effects are also taken into account but—unlike in the banana regime—the plateau transport is found to be unaffected by them. Although the strong electric field has practically no effect on the electron orbits due to their large thermal speed, the electron dynamics is indirectly affected, since the electron–ion collisions depend on the ion mean flow. The bootstrap current is evaluated by an adjoint method that relies on the knowledge of the Spitzer function which we approximate with a truncated Laguerre polynomial series. We find that it is unnecessary to keep more than two terms in this expansion to get reasonably accurate results. Of course, all of our results are consistent with conventional $U = 0$ plateau regime calculations.

As the electric field increases, the resonance causing the plateau transport, which would be at $v_{\parallel} \approx 0$ for $U = 0$, is now shifted toward the tail of the distribution. For a strong

electric field this leads to an exponential reduction in the ion heat flux for $U \gg 1$. However, for moderate values of U the ion heat diffusivity is enhanced [$L(|U| \approx 0.91) \approx 1.46$]. The shift of the resonance sets the upper limit of validity of the calculations presented at about $U = 3.5$ (we note that these higher normalized electric fields are unlikely in experiments).

The temperature gradient driven part of parallel ion flow—corresponding to the poloidal ion rotation—is multiplied by a U -dependent factor $J(|U|)$ that decreases until $J(|U| \approx 0.76) \approx 0.39$; then it starts to increase, approaching an asymptote of $2U^2 - 3$. The same factor appears also in the expressions for the poloidal impurity rotation and the bootstrap current multiplying the ion temperature gradient term. It modifies these quantities for experimentally relevant values of the radial electric field. In particular, finite- U modifications might account for the discrepancy found in recent experimental comparison with neoclassical predictions [14]. However, extremely precise measurements are required for quantitative comparisons in the plateau regime because of the flatness of the $J(|U|)$ curve below $|U| \sim 1.5$.

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Appendix A. Electron transport

The ion flow modified by the finite poloidal Larmor radius effects appears in the drift kinetic equation (33) through the electron–ion collision operator (35). Accordingly, we expect modifications in the electron transport. Starting with the kinetic equation for H_e (equations (37)–(40)), after the replacement $C_e^{(1)}(H_e) \rightarrow -\nu H_e$ (noting that the electron collision operator is not momentum conserving) we find that the equation to solve is

$$v\xi \partial_\theta H_e + \nu q R H_e = Q_e \sin \theta, \quad (\text{A1})$$

where we introduce $\xi = v_\parallel / v$ and

$$Q_e = \frac{\epsilon f_{\text{Me}} v^2}{T_e} \left\{ \frac{I(1 + \xi^2)}{2\Omega_e} \left[\frac{1}{n_e} \frac{\partial p}{\partial \Psi} + \left(\frac{mv^2}{2T_e} - \frac{5}{2} \right) \frac{\partial T_e}{\partial \Psi} \right] + \frac{B(1 - 3\xi^2)}{2} \left[\frac{J(U^2)}{2Z} \frac{IB}{\Omega_e \langle B^2 \rangle} \frac{\partial T_i}{\partial \Psi} - \frac{eE_\parallel}{v_e B} \sqrt{2} D(x_e) \right] \right\}, \quad (\text{A2})$$

and we use quasineutrality $n_e = Zn_i$. We obtain the plateau solution

$$H_e \approx \frac{Q_e}{v} \left(\pi \delta(\xi) \sin \theta - \frac{\cos \theta}{\xi} \right). \quad (\text{A3})$$

The particle transport is calculated from the electron version of (23) to find

$$\begin{aligned} \langle \Gamma_e \cdot \nabla \Psi \rangle &\approx - \left\langle \frac{I\epsilon}{2\Omega_e q R} \int d^3 v v^2 (1 + \xi^2) H_e \sin \theta \right\rangle = - \sqrt{\frac{\pi}{2}} \frac{I^2 \epsilon^2 n_e}{\Omega_e^2 q R_0} \left(\frac{T_e}{m_e} \right)^{3/2} \\ &\times \left\{ \frac{1}{p_e} \frac{\partial p}{\partial \Psi} + \frac{1}{2} \frac{\partial \ln T_e}{\partial \Psi} + \frac{J(U^2) T_i}{2 Z T_e} \frac{\partial \ln T_i}{\partial \Psi} - \frac{e E_1 \Omega_e}{I T_e v_e} \sqrt{2} \left(a_0 - \frac{a_1}{2} \right) \right\}. \end{aligned} \quad (\text{A4})$$

This expression reduces to the usual plateau result [3] in the $U \rightarrow 0$ and $E_1 \rightarrow 0$ limits.

Similarly, the electron heat flux is

$$\begin{aligned} \langle q_e \cdot \nabla \Psi \rangle &\approx - \left\langle \frac{I\epsilon}{2\Omega_e q R} \int d^3 v \frac{m_e v^4}{2} (1 + \xi^2) H_e \sin \theta \right\rangle - \frac{5 T_e}{2} \langle \Gamma_e \cdot \nabla \Psi \rangle \\ &= - \sqrt{\frac{\pi}{8}} \frac{I^2 \epsilon^2 p_e}{\Omega_e^2 q R_0} \left(\frac{T_e}{m_e} \right)^{3/2} \left\{ \frac{1}{p_e} \frac{\partial p}{\partial \Psi} + \frac{13}{2} \frac{\partial \ln T_e}{\partial \Psi} + \frac{J(U^2) T_i}{2 Z T_e} \frac{\partial \ln T_i}{\partial \Psi} \right. \\ &\quad \left. - \frac{e E_1 \Omega_e}{I T_e v_e} \sqrt{2} \left(a_0 - \frac{13 a_1}{2} \right) \right\}, \end{aligned} \quad (\text{A5})$$

where the Z dependent coefficients a_0 and a_1 from the Laguerre polynomial expansion of equation (B4) are given in equation (B9).

Appendix B. Spitzer function

Considering the ansatz given in equation (41) for the Spitzer problem defined by equation (36), the problem to solve is

$$C_e^{(1)} \left\{ - \frac{v_{\parallel}}{v_e} D(x_e) f_{Me} \right\} = v_{\parallel} f_{Me}, \quad (\text{B1})$$

where

$$C_e^{(1)} \{ \delta f \} = C_{ee}^{(1)} \{ \delta f \} + \mathcal{L}_{ei} \{ \delta f \} \quad (\text{B2})$$

is the linearized electron collision operator with $C_{ee}^{(1)} \{ \delta f \} = C_{ee} \{ \delta f, f_{Me} \} + C_{ee} \{ f_{Me}, \delta f \}$, C_{ee} is the full Fokker–Planck operator [3] and $\mathcal{L}_{ei} \{ \delta f \} = (3\sqrt{2}\pi/4) v_e Z (T_e/m_e)^{3/2} \nabla_v \cdot [\nabla_v \nabla_v v \cdot \nabla_v \{ \delta f \}]$ is the Lorentz operator.

An approximate solution to (B1) can be constructed variationally by maximizing the functional

$$\Lambda = \frac{v_e m_e}{2 p_e} \left[\int d^3 v \eta C_e^{(1)} \{ \eta f_{Me} \} - 2 \int d^3 v \eta v_{\parallel} f_{Me} \right], \quad (\text{B3})$$

where $\eta = -(v_{\parallel}/v_e) D(x_e)$ and we approximate the x_e dependent part of η by a truncated generalized Laguerre polynomial series expansion

$$D(x_e) \approx \sqrt{2} (a_0 L_0^{(3/2)}(x_e^2) + a_1 L_1^{(3/2)}(x_e^2) + a_2 L_2^{(3/2)}(x_e^2) + \dots). \quad (\text{B4})$$

By direct substitution of equation (B4) into equation (B3), using the orthogonality of the Laguerre polynomials $\int_0^{\infty} d\zeta e^{-\zeta} \zeta^{\alpha} L_j^{(\alpha)}(\zeta) L_k^{(\alpha)}(\zeta) = \delta_{jk} \Gamma(k + \alpha + 1)/k!$, we obtain

$$\Lambda = -2 \sum_{j,k} a_j a_k \left(\frac{M_{jk}^e}{\sqrt{2}} + Z M_{jk}^i \right) + \sqrt{2} a_0, \quad (\text{B5})$$

where we introduce the matrix elements [24]

$$M_{jk}^e = -\frac{\sqrt{2}}{v_e n_e} \int d^3 v x_{\parallel e} L_j^{(3/2)}(x_e^2) C_{ee}^{(1)}\{x_{\parallel e} L_k^{(3/2)} f_{Me}\} = \begin{pmatrix} 0 & 0 & 0 & \dots \\ 0 & 1 & \frac{3}{4} & \dots \\ 0 & \frac{3}{4} & \frac{45}{16} & \dots \\ \vdots & & & \ddots \end{pmatrix} \quad (\text{B6})$$

and

$$M_{jk}^i = -\frac{1}{v_e n_e} \int d^3 v x_{\parallel e} L_j^{(3/2)}(x_e^2) \mathcal{L}_{ei}\{x_{\parallel e} L_k^{(3/2)} f_{Me}\} = \begin{pmatrix} \frac{1}{2} & \frac{3}{4} & \frac{15}{16} & \dots \\ \frac{3}{4} & \frac{13}{8} & \frac{69}{32} & \dots \\ \frac{15}{16} & \frac{69}{32} & \frac{433}{128} & \dots \\ \vdots & & & \ddots \end{pmatrix}, \quad (\text{B7})$$

where $j, k = 0, 1, 2, \dots$. Keeping the first three terms in the expansion in equation (B3), maximization of Λ by using the solution of the equations $\partial \Lambda / \partial a_j = 0$ gives

$$a_0 = (576 + 1028\sqrt{2}Z + 434Z^2)/(8\sqrt{2}Z\gamma), \quad (\text{B8})$$

$$a_1 = -3(30 + 11\sqrt{2}Z)/(2\gamma),$$

$$a_2 = -3(4 - \sqrt{2}Z)/\gamma,$$

$$\gamma = 72 + 61\sqrt{2}Z + 16Z^2,$$

which is numerically equal to the result of [29]. If we neglect the third term in equation (B4) as well, the results are simply

$$a_0 = (8 + 13\sqrt{2}Z)/[4Z(2\sqrt{2} + 2Z)], \quad (\text{B9})$$

$$a_1 = -3/[2(2 + \sqrt{2}Z)].$$

Denoting the approximate Spitzer function keeping n polynomials in the Laguerre polynomial expansion by $f_S^{[n]}$, we can estimate the error between $f_S^{[2]}$ and $f_S^{[3]}$ for low Z by comparing some of their moments to those of $f_S^{[4]}$. For $Z = 1$ the relative error of a_0 (i.e. the error of the ohmic current) is around 1.5% for $f_S^{[2]}$ and 0.7% for $f_S^{[3]}$. The error of the pressure and ion temperature gradient parts of the bootstrap current [$\sim \int dx_e x_e^5 f_S(x_e)$] is less than 0.6% for $f_S^{[3]}$ ($Z = 1$) and happens to be even smaller for $f_S^{[2]}$.

In the $Z \rightarrow \infty$ limit, electron–electron collisions can be neglected compared with electron–ion collisions, and the Spitzer problem can be solved exactly. The result has the form of equation (41) with $D(x_e) = 4x_e^3/(3\sqrt{\pi}Z)$. In the high Z limit the relative error of the ohmic current remains below 4.5% for $f_S^{[2]}$, while it is less than 0.15% for $f_S^{[3]}$. The bootstrap current for $f_S^{[3]}$ has the same asymptotic limit as the exact solution; however, the form of the Spitzer function is different. The relative error for $f_S^{[2]}$ remains below 9%. The error introduced by the uncertainty of the Coulomb logarithm (that is the ultimate error of any Fokker–Planck theory) is usually higher than the error of the two-polynomial approximation (B9), so it seems unnecessary to keep three polynomials in the variational calculation of f_S for the plateau regime.

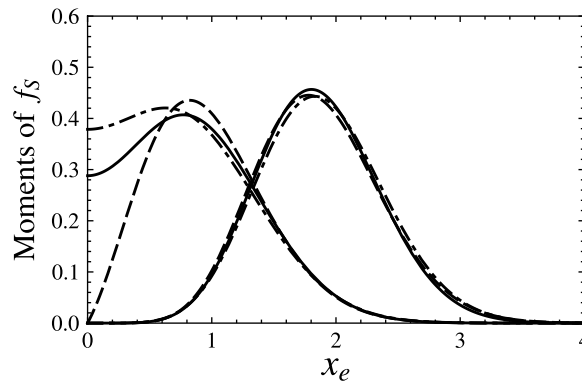


Figure 3. Different approximations of the Spitzer function for $Z = 1$. The curves with the maximum below $x_e = 1$ represent $e^{-x_e^2} D(x_e)$, the ones centered around $x_e = 2$ are $(x_e^5/4)e^{-x_e^2} D(x_e)$. Solid—three Laguerre polynomials, equation (B8) and [29] (not plotted separately since they would be indistinguishable). Dashed-dotted—two Laguerre polynomials, equation (B9). Dashed—fourth order polynomial, [30].

The Laguerre polynomial expansion contains only even powers of x_e . Thus they cannot reproduce the $\propto x_e^3$ behavior expected in the high Z limit. For this reason in [30] a simple fourth order polynomial approximation is constructed. Although this polynomial is correct asymptotically as $Z \rightarrow \infty$, the resulting Spitzer function is not smooth at $x_e = 0$ due to the non-vanishing linear term in the polynomial, as can be seen in figure 3. The figure shows four approximations of the Spitzer function, and the integrand of their fifth moment that appears in the bootstrap current calculation. The difference between the approximations of f_s is conspicuous for low values of the normalized velocity; however, the difference between the low-order (<10) moments of f_s —which have a physical relevance—is negligibly small.

References

- [1] Wagner F 2007 *Plasma Phys. Control. Fusion* **49** B1–33
- [2] Shimada M *et al* 2007 *Nucl. Fusion* **47** S1
- [3] Helander P and Sigmar D J 2001 *Collisional Transport in Magnetized Plasmas* (Cambridge: Cambridge University Press)
- [4] Connor J W, Fukuda T, Garbet X, Gormezano C, Mukhovatov V, Wakatani M, ITB Database Group and ITPA Topical Group on Transport and Internal Barrier Physics 2004 *Nucl. Fusion* **44** R1
- [5] Urano H, Takizuka T, Kamada Y, Oyama N, Takenaga H and Miura Y 2005 *Phys. Rev. Lett.* **95** 035003
- [6] Kim Y B, Diamond P H and Groebner R J 1991 *Phys. Fluids B* **3** 2050
- [7] Ernst D R, Coppi B, Scott S D, Porkolab M and TFTR Group 1998 *Phys. Rev. Lett.* **81** 2454
- [8] Terry P W 2000 *Rev. Mod. Phys.* **72** 109
- [9] Burrell K H *et al* 1994 *Phys. Plasmas* **1** 1536
- [10] Burrell K H 1999 *Phys. Plasmas* **6** 4418
- [11] Connor J W and Wilson H R 2000 *Plasma Phys. Control. Fusion* **42** R1
- [12] Parra F I and Catto P J 2009 *Plasma Phys. Control. Fusion* **51** 095008
- [13] Catto P J, Parra F I, Kagan G and Simakov A 2009 *Nucl. Fusion* **49** 095026
- [14] Marr K D, Lipschultz B, Catto P J, McDermott R M, Reinke M L and Simakov A N 2010 *Plasma Phys. Control. Fusion* **52** 055010
- [15] Solomon W M, Burrell K H, Andre R, Baylor L R, Budny R, Gohil P, Groebner R J, Holcomb C T, Houlberg W A and Wade M R 2006 *Phys. Plasmas* **13** 056116
- [16] Scarabosio A, Bortolon A, Duval B P, Karpushov A and Pochelon A 2006 *Plasma Phys. Control. Fusion* **48** 663
- [17] McDermott R M *et al* and the Alcator C-Mod Team 2009 *Phys. Plasmas* **16** 056103

- [18] Sips A C C for the Steady State Operation and the Transport Physics topical groups of the International Tokamak Physics Activity 2005 *Plasma Phys. Control. Fusion* **47** A19
- [19] Kagan G and Catto P J 2008 *Plasma Phys. Control. Fusion* **50** 085010
- [20] Galeev A A and Sagdeev R Z 1968 *Sov. Phys.—JETP* **26** 233
- [21] Hinton F L and Hazeltine R D 1976 *Rev. Mod. Phys.* **48** 239
- [22] Kagan G and Catto P J 2010 *Plasma Phys. Control. Fusion* **52** 055004
- [23] Rosenbluth M N, Hazeltine R D and Hinton F L 1972 *Phys. Fluids* **15** 116
- [24] Galeev A A 1970 *Zh. Eksp. Teor. Fiz.* **59** 1378
Galeev A A 1971 *Sov. Phys.—JETP* **32** 752
- [25] Hirshman S P and Sigmar D J 1981 *Nucl. Fusion* **21** 1079
- [26] Helander P 2001 *Phys. Plasmas* **8** 4700
- [27] Antonsen T M and Chu K R 1982 *Phys. Fluids* **25** 1295
- [28] Helander P and Catto P J 2001 *Phys. Plasmas* **8** 1988
- [29] Hirshman S P 1978 *Phys. Fluids* **21** 1295
- [30] Hirshman S P 1980 *Phys. Fluids* **23** 1238