Ion runaway in lightning discharges

T. Fülöp[1] and M. Landreman[2]

[1] *Department of Applied Physics, Nuclear Engineering, Chalmers University of Technology and Euratom-VR Association, Göteborg, Sweden*

[2] *Plasma Science and Fusion Center, MIT, Cambridge, MA, 02139, USA*

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Abstract

Runaway ions can be produced in plasmas with large electric fields, where the accelerating electric force is augmented by the low mean ionic charge due to the imbalance between the number of electrons and ions. Here we derive an expression for the high-energy tail of the ion distribution function in lightning discharges and investigate the energy range that the ions can reach. We also estimate the corresponding energetic proton and neutron production due to fusion reactions.

Introduction Strong electric fields have been observed in various plasmas, e.g. in connection with reconnection events in solar flares, disruptive instabilities in tokamaks or in lightning discharges in thunderstorms. If the electric field exceeds a certain limit, the accelerating force on the charged particles may overcome the friction on other species and they may be accelerated to very high energies. The phenomenon of electron runaway was described already in Ref. [1], where it was proposed that the strong electric field associated with thunderstorms could accelerate electrons to relativistic energies. Since then runaway electrons have often been observed and extensively studied.

The mechanism for ion runaway is different from electron runaway because the frictional drag due to the drifting electrons tends to cancel out the electric force and must be accounted for accurately, since the cancellation is complete in the simple case of a pure plasma. However, in certain circumstances (e.g. in the presence of high-Z impurities or magnetic trapping of electrons) ions can run away [2]. The phenomenon of runaway acceleration of ions in the presence of an electric field has been suggested to be present during magnetic reconnection events in tokamaks [3] and in solar flares [4, 5]. In this paper we investigate the possibility of ion runaway in lightning events in thunderstorms and its connection to neutron production. The main difference to the above mentioned cases is that in the circumstances characteristic of lightning events the plasma is only weakly ionized and the friction is often dominated by charge-exchange with neutral atoms instead of Coulomb collisions. However, as we will show, if there are more electrons than ions, the accelerating force is enhanced considerably and ions can be accelerated to keV energies.

There is a wealth of experimental observations that the strong electrical fields inside thunderclouds give rise to high-energy charged particles, γ-rays and neutrons [6]. The γ-rays are thought to be produced by relativistic electrons, in accordance with the relativistic runaway electron avalanche model that involves acceleration and multiplication of electrons [7]. These γ-rays can play a significant role in various physical processes, such as electron-positron pair production (which in turn may give rise to secondary runaway avalanches [8]) and neutron production due to photonuclear processes.

Neutron production correlated with lightning events has been reported on several occasions[9–15]. It has been shown that there is a correlation between the neutron flux and the thunderstorm electric field [16].The origin of the intense neutron fluxes has been debated. The first hypothesis was that neutrons are produced via deuterium fusion reactions [9]. Later, Babich and Roussel-Dupre [17] suggested that the fusion mechanism is not feasible under the usual physical conditions in lightning and proposed a photonuclear reaction ^{14}N(γ, n)^{13}N. The importance of the photonuclear mechanism for neutron production was confirmed by simultaneous γ-ray and neutron enhancements detected in thunderstorms [14]. However, although γ-rays in the energy range 10-100 MeV have been observed [18], to explain the neutron fluxes of the order of $(3-5)\times 10^2$ m^{-2}s^{-1}, reported in e.g. Ref. [13], the intensity of the γ radiation in the energy range 10-30 MeV should be several orders of magnitude higher than the one observed. Therefore the origin of neutron bursts associated with thunderstorms is not yet settled.

The purpose of the present work is to describe the phenomena of ion runaway in the context of lightning discharges. By solving the kinetic equation we will derive an expression for the high-energy tail of the ion distribution function and calculate the energies the ions can reach. We will illustrate the importance of the phenomena by studying one of the possible consequences: the enhanced neutron production due to the fact that the cross section for ^{2}H(^{2}H, n)^{3}He reaction (hereafter called DD reaction) is much larger if the ion energy lies in the keV instead of eV range.

Ion runaway The basic physics of conventional ion runaway in a pure plasma can be understood by considering a uniform unmagnetized plasma in equilibrium with an electric field. We may work in the ion rest frame, so the electrons have net motion antiparallel to the electric field E. Now consider a test charge, initially at rest. It experiences no net friction on the bulk ions because both the test charge and the bulk ions are at rest. The test charge experiences two other forces however: acceleration due to E, and friction with the moving electrons. If the test charge is positive, these forces will oppose each other. If the test charge has the same charge as the bulk ions, these two forces must be equal (and opposite), because otherwise the bulk ions would be accelerating. However, suppose the test charge has a different Z than the charge of bulk ions, Z_b. The forces scale differently with Z: electric field acceleration scales as Z, while friction on the drifting electrons scales as Z^2. Therefore, for $Z > Z_b$, the dominant force on the test charge will be electron friction, and so the charge will be dragged to high energies as its velocity equilibrates with the electron mean flow. For $Z < Z_b$, friction becomes unimportant, so the test charge accelerates along E as it would in vacuum.

The above physical description can be mathematically described (and generalized for a plasma with multiple ion species) in the following way. The equation of motion of an ion with mass m and charge Z in an ionized gas is given by

$$m\frac{dv}{dt} = eZE - \sum_j F_j - F_x \tag{1}$$

where v is the component of the velocity in the direction of the electric field, F_j is the drag force that the test ion experiences on the charged species j (electrons and other ion species) and F_x is the friction representing the interaction of the ion with the neutral atoms. When calculating the electron drag force, the distortion of the electron distribution function due to its drift in the imposed electric field must be taken into account. The electron drag force is [5]

$$F_e = F_{e0} + \frac{Z^2}{Z_i} C\left(\frac{v}{v_{Te}}\right) eE, \quad E \ll E_D$$

where F_{e0} is the drag force on the electrons at rest, $E_D = (n_e e^3 \ln\Lambda)/(4\pi\epsilon_0^2 T_e)$ is the Dreicer field, n_e and T_e are the electron density and temperature, respectively, $\ln\Lambda$ is the Coulomb logarithm and $Z_i = n_e^{-1}\sum_f n_f Z_f^2$ is the mean ionic charge. $C(x) = 1 - [2x^{-3}K_3(x) + K_0(x)]/K_0(\infty)$ and $K_n(x) = \int_0^x y^{n+4}\exp(-y^2)dy$. For $v/v_{Te} \ll 1$ it can be shown that $C \simeq 1$ and Eq. (1) can be rewritten as $m\,dv/dt = eZE^* - \sum_j F_{j0} - F_x$, where $E^* = E(1 - Z/Z_i)$ is the effective electric field. If $Z/Z_i = 1$, the effective field is zero and a test ion will always slow down. If the test ion charge is less than the mean ionic charge $Z < Z_i$, then the test ion sees an effective field in the same direction as the electric field E, which can be of nearly the same magnitude as E if $Z_i \gg Z$. On the other hand, if a test ion has $Z > Z_i$ it will experience an electric field E^* in the direction opposite to E, which can be larger than E and will cause the ion to accelerate in the direction of the electron streaming. In this case, the electron drag force exceeds the force exerted by the electric field and these ions are dragged along by the streaming electrons.

At least in some regions in thunderstorms and in some circumstances, it is thought that the electron density can significantly exceed the ion density, as shown in Fig. 3a of Ref. [19]. This departure from neutrality is consistent with Poisson's equation. Therefore we will assume that $n_i/n_e = Z_i \ll 1$ (leading to $Z/Z_i \gg 1$). We note however, that the exact value of Z/Z_i is not known, and in the following we will present the results for various values of Z/Z_i. This large factor Z/Z_i multiplies the accelerating electric force. Note that in this case, since $Z > Z_i$, the dominant accelerating force experienced by ions is friction with electrons, so the ions move antiparallel to the electric field.

A comprehensive understanding of which ions will be present in the lightning channel is lacking, partly because of the complexity of the chemical reactions and the variety of the species involved [20]. For simplicity, here we will only consider acceleration of deuterium ions. Assuming charge-exchange interaction dominates the friction due to the neutral atoms we can approximately write $F_x = mn_a\sigma_x v^2$, where σ_x is the charge-exchange cross-section, and n_a is the number density of the neutral atoms. We are interested in the charge-exchange cross-section of deuterium ions and atmospheric constituents (mainly nitrogen and oxygen), σ_x^D. Fit formulas for σ_x for collisions between H^+ with H, O, N_2, O_2 can be found in Ref. [21]. As we are only interested in an order of magnitude approximation, in this work we will approximate σ_x^H with the formula obtained for collisions between H^+ and N_2: $\sigma_x^H = a_1(e^{-(\ln E_k - a_2)^2/a_3})(1-e^{-E_k/a_4})^2 + (a_5 - a_6\ln E_k)^2(1-e^{-a_7/E_k})^2 10^{-20}\ \mathrm{m}^2$, where $a_{1,2,..} = \{12.5, 1.52, 3.97, 0.36, -1.2, 0.208, 0.741\}$ and E_k is the projectile (H^+) energy in keV. For the energies of interest no isotope effect is expected [22], and the charge-exchange cross-section for collisions between deuterium and N_2 can be obtained by rescaling the cross-section for protons: $\sigma_x^D(E_k) = \sigma_x^H(E_k/2)$. The result is shown in Fig. 1, together with a simpler fit $\sigma_x^D = (5/2)E_k/(1+(0.1E_k)^2)10^{-20}\ \mathrm{m}^2$. Note that the curve for deuterium has a maximum at around 10 keV.

Even if in most circumstances charge-exchange friction dominates, for completeness we also take into account the friction due to Coulomb collisions. The drag force on a test particle with charge number Z moving in a Maxwellian distribution function of charged field particles with charge number Z_f is $F_f \simeq (n_f Z^2 Z_f^2 e^4 \ln\Lambda)/(4\pi\epsilon_0^2 T_f)\,(1+m_f/m)\,G\,(v/v_{Tf})$, where m_f, T_f and n_f are the mass, temperature and number density of the field particles and $G(x)$ is the Chandrasekhar function. Thus, the total friction acting on the test ion should in general be a non-monotonic function of velocity. The condition for the ions to be accelerated is

$$\frac{E}{E_D} > \frac{\sum_j F_{j0} + F_x}{(Z|1 - Z/Z_i C(v/v_{Te})|)eE_D}. \tag{2}$$

Figure 2 shows the normalized drag force (from the right hand side of the Eq. (2)) as

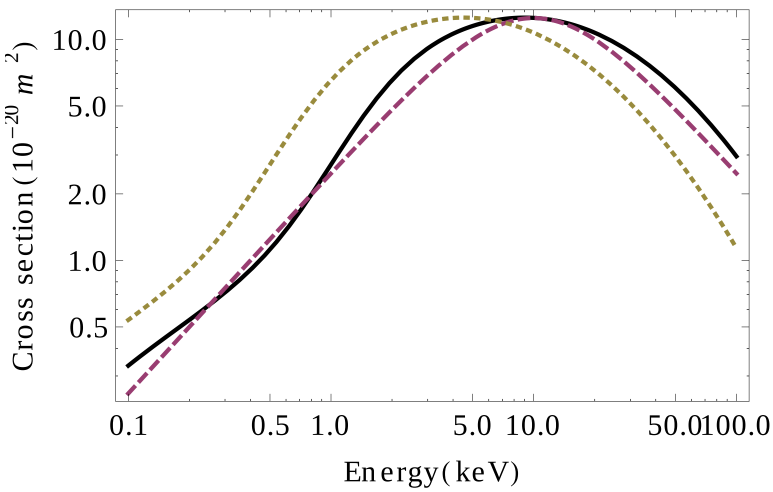

FIG. 1. Cross sections for charge exchange of D^+ with N_2 as a function of projectile energy. Dashed line shows the fit $\sigma_x^D = (5/2)E_k/(1 + (0.1E_k)^2)10^{-20}$ m^2 and dotted line shows the cross-section for charge exchange of H^+ with N_2.

a function of ion speed normalized to the thermal electron speed for deuterium ions, assuming 78% N_2^+ and 21% O_2^+. Since the value of the mean ionic charge and the electron density is unknown, we present results for Z_i ranging from 10^{-3} to 10^{-1} and n_e ranging from 10^{21} m^{-3} to 10^{23} m^{-3}. The number of neutral atoms is assumed to be $n_a = 10^{25}$ m^{-3} and the electron temperature $T_e = 3$ eV. Figure 2 shows that for these parameters, the deuterium ions can be accelerated to several tens of percent of the electron thermal speed, e.g. to keV energies.

If the friction due to Coulomb collisions is neglected we can estimate the energy of the ions when the accelerating force and the charge exchange friction is equal. This gives $T_i \equiv mv^2/2 = Z^2eE/(2Z_in_a\sigma_x)$. As σ_x decreases for high velocities, for sufficiently high values of $E/(Z_in_a)$, ions can run away to arbitrarily high energy. For protons this is expected to happen for $E_{kV/m}/(Z_in_a^{20}) \gtrsim 150$, where $E_{kV/m}$ is the electric field in kV/m and $n_a^{20} = n_a10^{-20}$. If the electric field is not larger than this value, we will still have acceleration of ions, and the energy is determined by the solution of $T_i = Z^2eE/(2Z_in_a\sigma_x)$, as shown in Fig. 3.

The consequence of this is that deuterium ions should have energies in the keV

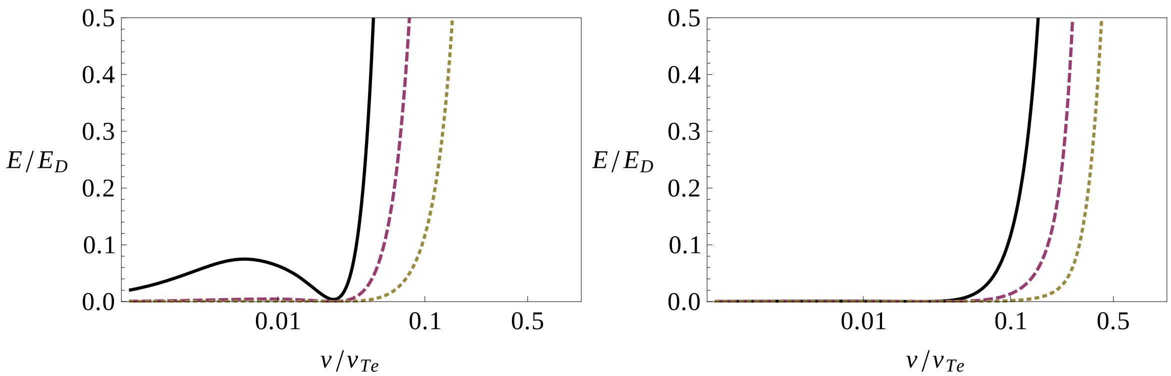

FIG. 2. E/E_D from Eq. (2) as a function of the deuterium ion speed normalized to the electron thermal speed. (a) The electron density is $n_e = 10^{21}$ m^{-3}. Solid is for mean ionic charge $Z_i = 0.1$, dashed is for $Z_i = 0.01$ and dotted is for $Z_i = 0.001$. (b) The mean ionic charge is $Z_i = 0.001$. The electron density is $n_e = 10^{21}$ m^{-3} (solid), $n_e = 10^{22}$ m^{-3} (dashed), $n_e = 10^{23}$ m^{-3} (dotted).

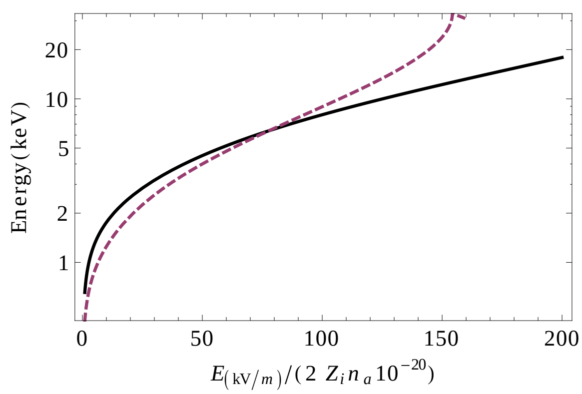

FIG. 3. Ion energy obtained as the solution of $T_i = Z^2 eE/(2Z_i n_a \sigma_x)$. Solid line is for deuterium ions, dashed line is for protons.

range where the DD fusion reaction cross section is not negligible. Note that even if the deuterium ions are neutralized due to charge-exchange eventually, the neutral deuterium will continue to have high energy and have a chance to fuse with other deuterium (neutral or ionized). The fusion reaction cross-section is also expected to be enhanced compared to the bare nuclei case due to electron screening [23].

To determine the high-energy tail of the ion distribution function we can solve the kinetic equation for the ions, which in this case is $0 \simeq C_{ie} + C_{in}$, where C_{in} represents ion-neutral friction. The ion-electron friction can be written as $C_{ie} = -(F_{ie}/m)(\partial f/\partial v_\parallel)$ (Eq. (3.65) in [25]), where F_{ie} is the ion-electron friction. Assuming the electron energy is determined by a balance between the electric field and electron-ion friction, rather than by another process such as electron-neutral friction, then $F_{ie} = (Z^2 eE/mZ_i)$. In this case the kinetic equation becomes $(Z^2 eE/mZ_i)(\partial f/\partial v_\parallel) = -n_a \sigma_x |v_\parallel| f$, where $v_\parallel$ is the velocity component along the electric field. Using $\sigma_x = 2.5E_k/(1+(0.1E_k)^2)10^{-20}$ m^2, we obtain the following solution for the distribution function:

$$f(v_\parallel) = C_f(1 + Bv_\parallel^4)^{-A/(4B)}, \tag{3}$$

where $A = (5/4)(n_a/EZ_i)(m/Ze)^2 10^{-23}$ and $B = (10^{-4}m/2e)^2$. Assuming the ion distribution can be written in the form $f = C_f \delta(v_x)\delta(v_y)\Theta(v_\parallel)(1 + Bv_\parallel^4)^{-A/(4B)}$, where δ is the Dirac delta function and Θ is a Heaviside step function, we can determine C_f by requiring $n_i = \int d^3 v f$, where n_i is the ion density. This leads to $C_f = n_i / \int (1 + Bv^4)^{-A/(4B)} dv$.

As long as there is an electron population with a mean speed U that is much higher than the ion speed, ions can also be accelerated by ion-electron friction even without the presence of an external electric field. The ion-electron collision operator can be derived by expanding the Rosenbluth-potential form of the full Fokker-Planck operator given by Eq. (3.21) in Ref. [25], assuming the electron velocity U is much larger than the ion velocity. The resulting operator is $C_{ie} = (n_e e^4 \ln\Lambda)/(4\pi\epsilon_0^2 m_i m_e U^2)(\partial f_i/\partial v_\parallel)$, yielding the kinetic equation $0 \simeq (n_e e^4 \ln\Lambda)/(4\pi\epsilon_0^2 m_i m_e U^2)(\partial f_i/\partial v_\parallel) - n_a \sigma |v_\parallel| f_i$. This kinetic equation is identical to that in the previous paragraph but with a different A. In this form, the quantities that need to be specified are the electron density n_e and beam speed U.

Neutron production As an example of an application we present an estimate of the amount of neutrons that can be produced by fusion process mechanisms, highlighting the role of the ion runaway process. The neutron production from the DD reaction can be estimated as $N_n = n_D V_{ch} \Delta t \int_0^\infty v f \sigma_{DD} d^3 v$, where V_{ch} is the volume of the lightning channel, n_D is the number density of deuterium, v and f is the velocity and the distribution function of the deuterium ions, Δt is the lifetime of the electric field within the lightning channel and σ_{DD} is the cross section for the nuclear fusion reaction. Using the expression for the DD fusion cross-section given in Ref. [24], we can estimate the number of neutrons by $N_n = n_D V_{ch} \Delta t n_i \int_0^\infty dv v (1 + Bv^4)^{-A/4B} \sigma_{DD} / \int_0^\infty dv (1 + Bv^4)^{-A/4B}$. The volume of the lightning channel, the lifetime of strong electric field and density of deuterium ions varies several orders of magnitude from case to case. The volume of the lightning channel can be assumed to vary between $V_{ch} \simeq 0.1 - 10$ m^3 [17]. The lifetime of the electric field Δt is approximated with the typical lifetime of the return stroke $\Delta t \simeq 50$ μs in Ref. [17], but in some circumstances, as described in e.g. Ref. [15] quasistable electric fields, lasting tens of minutes can be formed during the mature stage of thunderstorms. Therefore, Δt can vary by many orders of magnitude from 5×10^{-5} s to several hundred seconds. The density of deuterium depends on the relative concentration of water vapour in thunderstorm (estimated to be 1.65% of the density of H_2O [17]) and relative fraction of deuterium in naturally occurring hydrogen (0.0156%), and the temperature and pressure conditions at the altitude of interest. It can be approximated to be $n_D \simeq 10^{24} \times 0.0165 \times 2 \times 0.00015 \simeq 5 \times 10^{18}$ m^{-3}. The number density of the deuterium ions should be lower than this value. Figure 4 shows the number of neutrons as a function of $E/(n_a Z_i)$ for a lightning channel containing deuterium ions with the density $n_i = 10^{18}$ m^{-3}. According to the figure, the number of neutrons that are produced by DD fusion reactions is substantial, and could be part of the explanation for the high neutron fluxes reported in e.g. in Ref. [13].

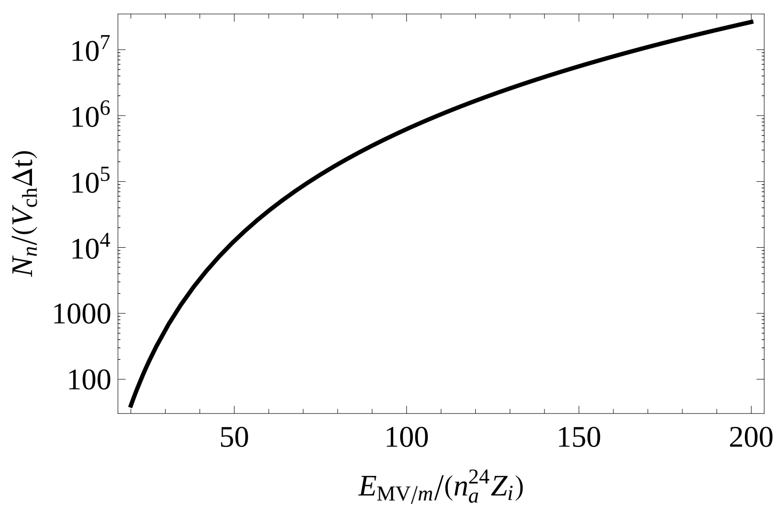

FIG. 4. Number of neutrons as function of $E/(n_a Z_i)$, where E is in MV/m and n_a is in units of 10^{24} m^{-3}.

If we would assume that the charge-exchange cross section is nearly independent of velocity, the solution would be $f = C_f \exp(-x^2)$, where $x = v_\parallel / v_{\mathrm{eff}}$, $v_{\mathrm{eff}} = \sqrt{2T_{\mathrm{eff}}/m}$ with

$T_{\rm eff} = eE/(Z_i n_a \sigma_x)$. This result would be in agreement with to the solution obtained in Ref. [17] except for the factor $1/Z_i$ in the expression for the effective temperature $T_{\rm eff}$. However, due to the non-monotonic dependence of the charge-exchange cross-section on velocity, the distribution function exhibits an elevated tail as given in Eq. (3). The combined effect of the factor $1/Z_i$ and the different form of the distribution function is the reason for the substantial neutron production compared to the estimates in Ref. [17].

Note that apart from the neutrons, the DD fusion reactions also produce energetic protons, tritons and ^{3}He. These particles are born with MeV energies and are likely to run away (as their initial energy is beyond the peak of the charge-exchange cross-section) and may engage in various neutron-producing fusion and fission reactions, such as ^{2}H(^{3}H,n)^{4}He and ^{15}N(^{1}H,n)^{15}O. As the number of the energetic ions produced in this way is not substantially larger than the number of neutrons calculated above, these secondary reactions are only expected to contribute with approximately the same amount of neutrons. However, the presence of large amounts of protons with energies of several MeV should be interesting for the understanding of the lightning initiation itself.

In conclusion we find that ions are likely to reach very high velocities in lightning discharges. The energy of the ions can be estimated by balancing the accelerating ion-electron friction force and the charge-exchange friction to obtain $T_i \simeq eZ^2E/2Z_i n_a \sigma_x$. From this result it can be seen that charge imbalance between electrons and ions, leading to a mean ionic charge $Z_i = n_i/n_e$ that is much less than one, can lead to very high ion energies. The process gives rise to an elevated tail in the ion distribution function with several interesting consequences, e.g. substantial neutron and proton production via DD fusion reactions. The phenomena of ion runaway may be of importance also in other circumstances where the mean ionic charge is different than one simultaneously with the presence of large electric fields, such as in laser-plasma based particle accelerators.

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