Mode conversion of waves in the ion cyclotron frequency range in magnetospheric plasmas

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Waves in the ion cyclotron range of frequencies with linear polarization detected by satellites can be useful for estimating the heavy ion concentrations in planetary magnetospheres. These waves are considered to be driven by mode conversion (MC) of the fast magnetosonic waves at the ion-ion hybrid resonances. In this paper, we derive analytical expressions for the MC efficiency and tunneling of waves through the MC layer. We evaluate the particular parallel wavenumbers for which MC is efficient for arbitrary heavy ion/proton ratios and discuss the interpretation of the experimental observations.

Introduction The presence of multiple ion species influences the plasma dispersion characteristics in various magnetized plasmas. In fusion plasmas, contamination with carbon impurities was shown to have profound consequences on the absorption efficiency of ion cyclotron resonance heating and can be used for its optimization [1]. Waves in the ion cyclotron range of frequencies, further referred to as ion cyclotron waves, have been frequently observed and studied also in planetary magnetospheres. Observational studies indicate, in addition to protons, the presence of several ion species e.g. He^{+} at the Earth's, Na^{+} at Mercury's, S^{2+} and O^{+} at Jupiter's planetary environments [2]. The presence of multiple ion species, even with small concentrations, can lead to the appearance of new and modified resonance, cutoff and crossover frequencies [3]. The observation of these waves can give information on the concentration of the various ion species and the underlying physical processes. The waves can be important in coupling different ion species via wave-particle interactions. For example, in the terrestrial magnetosphere it is believed that ion cyclotron waves, excited by energetic protons, transfer energy to helium (and maybe even oxygen) ions which escape from the ionosphere [4]. Ion cyclotron wave interactions are also considered as an important non-adiabatic process leading to pitch-angle scattering of protons.

The frequency of ion cyclotron waves in magnetospheric plasmas is usually within the ultra-low frequency (ULF) range (a few Hz) due to the low magnetic field of the planets. Narrowband linearly polarized ULF waves, clearly having a resonant structure, have been observed in the magnetospheres of the Earth and Mercury [5, 6]. Such ULF events have been suggested to be driven by mode conversion (MC) of the fast magnetosonic waves (FW) at the ion-ion hybrid (IIH) resonances [7]. The radial inhomogeneity of the planetary magnetic field causes the presence of a cutoff layer in the direction of lower magnetic fields close to the resonance layer. The efficiency of mode conversion and tunneling of waves through the MC layer formed by the cutoff-resonance pair depends on the ion composition. This gives restrictions on the heavy ion concentrations.

In this paper we discuss how tunneling and MC efficiencies depend on the concentrations of various ion species and the FW parallel wavenumber. We show that for strictly perpendicular wave propagation from the outer magnetosphere MC can only be efficient for very small heavy ion concentrations, i.e. less than a few percent. But for such low concentrations the observed ULF frequency would be practically indiscernible from the heavy ion cyclotron frequency, in contradiction to the observations. On the contrary, we will show that for particular finite parallel wavenumbers, MC can be efficient for arbitrary heavy ion/proton density ratios. This happens when the IIH resonance approaches the cutoff layer and for such conditions the IIH frequency is close to the crossover frequency. The results can be used for the interpretation of observations of linearly polarized waves in the magnetospheres and to estimate the local heavy ion concentration. Similar diagnostic techniques based on launching the FW in plasmas and further analysis of the MC and reflected waves have recently become a tool to measure the ion composition also in fusion plasmas [8, 9].

Wave dispersion To analyze how efficiently the waves coming from the outer magnetosphere tunnel through the IIH resonance and penetrate into the inner magnetosphere, we adopt a simplified 1D slab model that captures the essential features of the IIH resonance and MC process [2, 7]. The dispersion relation for the fast magnetosonic wave propagating predominantly across the magnetic field lines is approximately given by [10]:

$$n_{\perp,\mathrm{FW}}^2 = \frac{(\epsilon_\mathrm{L} - n_\parallel^2)(\epsilon_\mathrm{R} - n_\parallel^2)}{\epsilon_\mathrm{S} - n_\parallel^2} = \frac{(\epsilon_1 - n_\parallel^2)^2 - \epsilon_2^2}{\epsilon_1 - n_\parallel^2}, \tag{1}$$

where $n_\parallel = ck_\parallel/(2\pi f)$ is the refractive index parallel to the equilibrium magnetic field, and f is the wave frequency. In Eq. (1), $\epsilon_\mathrm{S} = \epsilon_1$, $\epsilon_\mathrm{L} = \epsilon_1 - \epsilon_2$ and $\epsilon_\mathrm{R} = \epsilon_1 + \epsilon_2$ are the plasma dielectric tensor components in the notation of Stix [10]. In a cold-plasma limit and for the ion cyclotron frequency range the tensor components can be written as

$$\epsilon_1 = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \sum_i \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2}, \quad \epsilon_2 = -\sum_i \frac{\omega}{\omega_{ci}} \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2}, \tag{2}$$

where the summation is to be taken over all ion species constituting the plasma, and $\omega_{pe,i}$ and $\omega_{ce,i}$ are the plasma and cyclotron frequencies of electrons and plasma ions, respectively.

The resonance condition $\epsilon_\mathrm{S} = n_\parallel^2$ for waves in the ion cyclotron frequency range is commonly identified with the IIH resonance [2, 10]. This resonance arises in plasmas which include at least two ion species with different charge-to-mass ratios. Full-wave treatment of the problem resolves this resonance and bends it into a confluence such that at this layer the FW is converted to a short wavelength mode. As Fig. 1(a) illustrates, when the inhomogeneity of the magnetic field is accounted for, the IIH resonance is accompanied by the L-cutoff layer ($\epsilon_\mathrm{L} = n_\parallel^2$) to the low magnetic field side, where the incoming FW undergoes partial reflection. The IIH resonance and L-cutoff layers form together the MC layer ($R_\mathrm{S} < R < R_\mathrm{L}$), which is a barrier for the propagating FW since within this region $n_{\perp,\mathrm{FW}}^2 < 0$. The FW power transmits through the MC layer via tunneling.

The equations for the IIH resonance, ω_S, and L-cutoff, ω_L, frequencies can be rewritten in a simpler form [11]

$$\sum_i \frac{f_i \mathcal{Z}_i}{\mathcal{Z}_i^2 - \tilde{\omega}_\mathrm{S}^2} = \alpha, \sum_i \frac{f_i}{\mathcal{Z}_i - \tilde{\omega}_\mathrm{L}} = \alpha, \tag{3}$$

where for individual ion species 'i' with the charge number Z_i and atomic mass A_i the following notations were introduced: $X_i = n_i/n_e$ – concentration of ion species, $f_i = Z_i X_i$ – fraction of the replaced electrons, $\mathcal{Z}_i = Z_i/A_i$. Throughout the paper the tilde sign over the frequency indicates its normalization to the cyclotron frequency of hydrogen ions ($\tilde{\omega} = \omega/\omega_{c\mathrm{H}}$). The parameter α entering the right-hand side of Eqs. (3) describes the effect of the finite $k_{\|}$ on the location of the resonance and cutoff in a plasma, and is given by $\alpha = (\omega_{\rm cH}^2/\omega_{\rm pH}^2)\,\tilde{n}_{\|}^2$, where $\omega_{p\rm H} = \sqrt{4\pi n_e e^2/m_{\rm H}}$ and $\tilde{n}_{\|}^2 = n_{\|}^2 - 1 - \omega_{pe}^2/\omega_{ce}^2$.

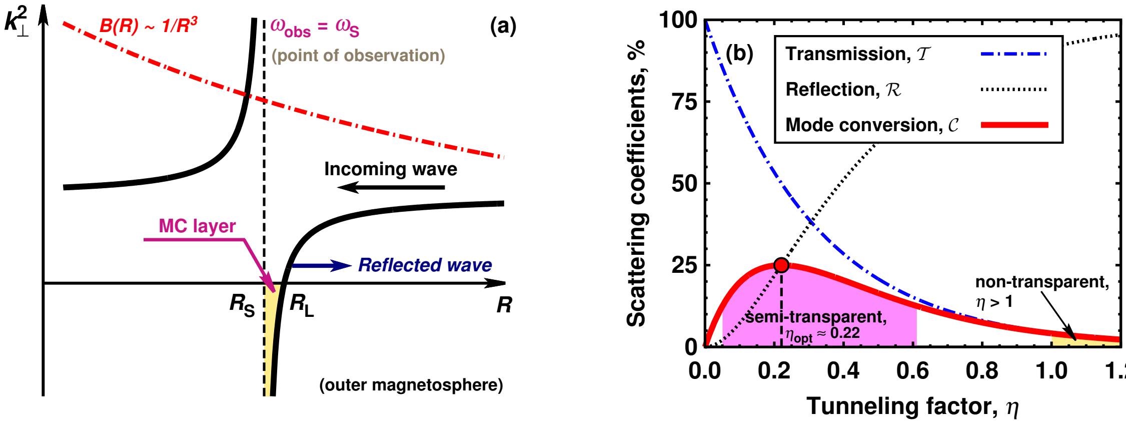

FIG. 1: (a) Sketch of the wave dispersion: due to the radial inhomogeneity of the magnetic field the IIH resonance is accompanied by a cutoff layer at the low field side, which form together the MC layer. (b) Scattering coefficients as a function of the tunneling factor for the isolated MC layer for waves incident from the outer magnetosphere.

Efficiency of mode conversion A quantitative parameter describing transmission through the MC layer and how much incident power undergoes mode conversion is referred to as the tunneling factor [12]. It is defined as

$$\eta = \frac{2}{\pi}\int_{R_{\rm S}}^{R_{\rm L}} |k_{\perp,\rm FW}(R)|\, dR, \tag{4}$$

where $k_{\perp,\rm FW} = (2\pi f/c)n_{\perp,\rm FW}$. The transmission coefficient is then $\mathcal{T} = \mathrm{e}^{-\pi\eta}$, which is independent of the wave incidence side. However, the reflection and thus MC coefficients are essentially different, when the wave approaches the MC layer from the resonance (inner magnetosphere), compared to when it comes from the cutoff (outer magnetosphere) side. For the former case the reflection is negligible and most of the FW energy undergoes mode conversion if the tunneling factor is large, $\eta \gg 1$. For the latter case, when the waves come from the outer magnetosphere (from the cutoff side), there is a reflection of the wave with the reflection coefficient $\mathcal{R} = (1-\mathcal{T})^2$, limiting the conversion coefficient to be $\mathcal{C} = \mathcal{T}(1-\mathcal{T})$.

Resonant absorption via MC can only be efficient if the layer is semi-transparent. Absorption reaches its maximum if the tunnelling factor $\eta = \ln 2/\pi \approx 0.22$. If $\eta \gg 1$ most of the power is reflected at the L-cutoff and cannot tunnel through the layer, making MC highly inefficient. If $\eta \ll 1$, the MC coefficient is also low since most of the power tunnels through the layer. Figure 1(b) shows that the mode conversion is most efficient if the tunneling factor has a value $0.05 < \eta < 0.61$. Under these conditions, the conversion coefficient is greater than a half of its maximum value, $\mathcal{C} \geq 0.5\,\mathcal{C}_{\max}$.

The tunneling factor can be calculated by expanding the dielectric tensor components at the IIH resonance, $R = R_{\rm S}$: $\epsilon_1(R) = n_\parallel^2 + \epsilon_1'(R - R_{\rm S})$, $\epsilon_2(R) = \epsilon_2 + \epsilon_2'(R - R_{\rm S})$, where the prime denotes derivative with respect to R. Then, the dispersion equation for the FW can be rewritten as follows:

$$k_{\perp,\,\rm FW}^2(R) = -\frac{\omega^2}{c^2}\frac{2\epsilon_2\epsilon_2'}{\epsilon_1'}\left[1 + \frac{\epsilon_2}{2\epsilon_2'(R - R_{\rm S})}\right] = k_{\rm A}^2\left[1 - \frac{\Delta}{R - R_{\rm S}}\right], \tag{5}$$

which is equivalent to the Budden potential [10, 13] with the asymptotic FW perpendicular wavenumber $k_{\rm A} = (\omega/c)\sqrt{-2\epsilon_2\epsilon_2'/\epsilon_1'}$ and the MC layer width $\Delta = -\epsilon_2/(2\epsilon_2')$. For the Budden problem the tunneling factor is given by

$$\eta = k_{\rm A}\Delta = (\omega/c)\sqrt{-\epsilon_2^3/(2\epsilon_1'\epsilon_2')}\,, \tag{6}$$

thus to evaluate the tunneling factor, the quantities ϵ_1', ϵ_2 and ϵ_2' have to be calculated at the IIH resonance.

For $k_\parallel = 0$ and a two-ion species plasma, using the tensor components and their derivatives at the resonance [12], Eq. (6) leads to $\eta_{10} = [\omega_{p\rm H}L_{\rm B}/(2c)]\sqrt{Z_2/A_2}[(1-\mu)^2/\mu^2]\sqrt{(1+\mu)/2}\;\; f_2(1-f_2)\sqrt{a_1/[a_2^3(1-g_1)]}$, where $\mu = q_1m_2/(q_2m_1)$ (subscripts '1' and '2' denote majority and minority ions, respectively), $a_1 = 1-(1-\mu)f_2$, $a_2 = 1-(1-1/\mu)f_2$, $g_1 = (1+\mu)(1-1/\mu)^2 f_2(1-f_2)/(2a_2)$. Here, $L_{\rm B} = -(\partial \ln B/\partial R)^{-1} = R_{\rm S}/N_B$ is the characteristic magnetic field gradient length, $R_{\rm S}$ is the radial position of the IIH resonance (point of observation) and N_B is an exponent describing the magnetic field variation and defined as $B(R) = B_0(R_0/R)^{N_B}$. The planetary magnetic field has a dipole structure best described by $N_B = 3$.

When estimating the tunneling factor using an expansion of the tensor components at the resonance, the dispersion of the FW, Eq. (1), can be fitted accurately by Eq. (5) at $R = R_{\rm S}$. However, as Eq. (4) shows, there is a finite contribution to the tunneling factor from the whole MC layer, although the contribution from the cutoff area is smaller than that from the area close to the resonance. To account for that, we derive a more accurate formula for the tunneling factor. We replace ϵ_2' in Eq. (6) with $-\epsilon_2/(2\Delta)$, and keep a rigorous expression for the MC layer width, $\Delta_{\rm MC}/R_s = (a_1 a_2)^{1/(2N_B)} - 1$, instead of its Budden approximation. Then, the tunneling factor, $\eta_2 = (\omega/c)(\epsilon_2^2\,\Delta_{\rm MC}/\epsilon_1')^{1/2}$ as a function of minority concentration and plasma parameters for $k_\parallel = 0$ is given by

$$\eta_{20} = \left(\frac{\omega_{\rm pH} L_{\rm B}}{2c}\right)\sqrt{\frac{Z_2}{A_2}}\,\frac{(1-\mu)^2}{\mu^{3/2}}\; f_2(1-f_2)\; G(f_2, N_B), \qquad G(f_2, N_B) = \sqrt{\frac{2N_B\, a_1}{a_2^2}\cdot\frac{(a_1 a_2)^{1/(2N_B)} - 1}{a_1 a_2 - 1}}\,. \tag{7}$$

The function $G(f_2, N_B)$ tends to unity for small minority concentrations. Fig. 2(a) shows a comparison of the tunneling factor calculated as a function of He^+ concentration at the geostationary orbit, $R = 6.6 R_{\rm E}$. Circles represent the tunneling factor calculated by numerical evaluation of the general expression Eq. (4), whereas dashed and dash-dotted lines represent analytical approximations given by η_{10} and η_{20}. The approximation for the tunneling factor given by η_{20} is in very good agreement with the numerical values, while the approximation only involving tensor components at the resonance differs by a factor of $\sim (2\mu/(1+\mu))^{1/2}$. Note that η_{10} and η_{20} are valid for arbitrary f_2, and obey $\eta(f_2,\mu) = \eta(1-f_2, 1/\mu)$. In Ref. [5], $f_{\rm obs}/f_{\rm cH} \approx 0.44$ was reported, and then it corresponds to $X[{\rm He}^+] = 38.9\%$. For such a concentration of helium ions $\eta_{20} = 24.6$, and the MC efficiency is completely negligible, as $\mathcal{C} \approx \mathcal{T} \approx 2.7\times 10^{-34}$. As Fig. 2(b) clearly illustrates, MC can be efficient only for He^+ concentrations less than a few percent. Almost total reflection ($\eta = 1$) is already reached at $X[{\rm He}^+] = 5.5\%/1.8\%/0.6\%$ for plasma densities $n_e = 1/10/100\,{\rm cm}^{-3}$, respectively. However, for low helium concentrations the IIH frequency is very close to the He^+ cyclotron frequency, and thus contradicts the observations.

Critical parallel wavenumber Now we will show how finite wave tunneling can occur for large minority concentrations and the resonant IIH frequency can be consistent with the observed values. As $k_\parallel$ increases, both the IIH resonance and L-cutoff frequencies increase (as shown in Fig. 3) and the corresponding layers shift towards the low magnetic field side; at the same time the distance between the layers gradually decreases. For a certain parallel wavenumber, $k_\parallel = k_\parallel^*$ (further referred to as a critical wavenumber), the MC layer width approaches zero, and thus the tunneling may be significant. When $\omega_{\rm S} = \omega_{\rm L}$, we have $\epsilon_2 = 0$, and this condition determines the crossover frequency, $\omega_{\rm cr}$ [14].

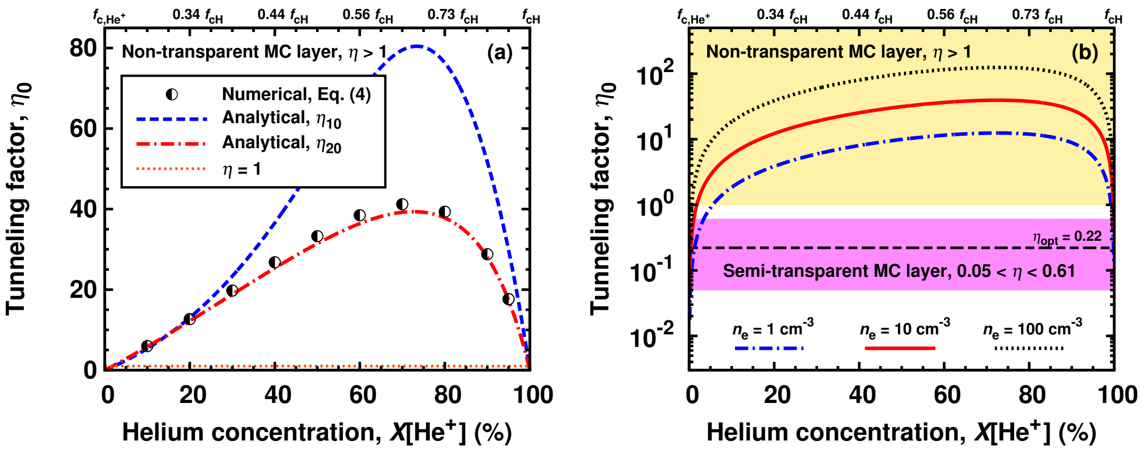

FIG. 2: (a) Tunneling factor as a function of the He^+ concentration at the geostationary orbit for $k_\parallel = 0$, $n_e = 10\,\mathrm{cm}^{-3}$. (b) Tunneling factor η_{20} (log-scale) for various plasma densities. Note that in all cases, MC can only be efficient for $X[\mathrm{He}^+]$ less than a few percent.

In two-ion component plasmas it is given by $\omega_{\mathrm{cr}}/\omega_{c2} = \sqrt{1-(1-\mu^2)f_2}$, and is higher than both ω_{L0} and ω_{S0}. Using Eqs. (3), it can be shown that $\omega_{\mathrm{S}} = \omega_{\mathrm{L}} = \omega_{\mathrm{cr}}$ is fulfilled if the parameter α is equal to $\alpha^* = 1/(\mathcal{Z}_1 + \mathcal{Z}_2)$. This yields the FW critical parallel wavenumber to be

$$k_\parallel^* = \frac{\omega_{p\mathrm{H}}}{c} \frac{(f/f_{\mathrm{cH}})}{\sqrt{\mathcal{Z}_1 + \mathcal{Z}_2}} \,. \tag{8}$$

For parallel wavenumbers close to the critical wavenumber but somewhat smaller, the fraction of the MC power for the waves incident from the outer magnetosphere (low magnetic field side) can be large. This is due to the significant reduction of the MC layer width, that may result in smaller values for the tunneling factor. We therefore conclude that $k_\parallel$ must be sufficiently large in order to explain the observations. Note that the ratio $k_\parallel^*/k_\perp \propto (R_\mathrm{S}/R)^{N_B}$ is small at the outer magnetosphere regions far away from the MC layer, whereas within the cutoff-resonance layer it varies strongly.

Estimate of the heavy ion concentration We are now in a position to discuss the implications of these results for determining the heavy ion concentrations in planetary magnetospheres. The main constituents of Mercury's magnetosphere are H^+, He^+ and Na^+. The source of H^+ and He^+ is the solar wind, while other heavy ions (Na^+, O^+ and K^+) result from sputtering of the planetary surface. The contributions of O^+ and K^+ are negligible compared to that of Na^+, that makes up between 10–50 % of the magnetospheric ion composition [7, 14]. The presence of Na^+ is therefore important

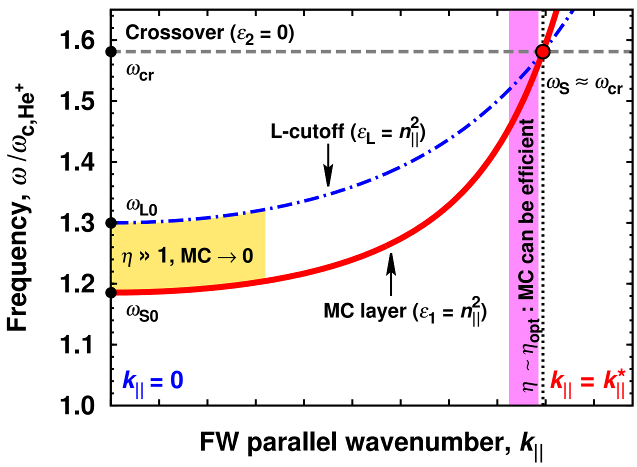

FIG. 3: Normalized IIH resonance and L-cutoff frequencies as a function of the FW parallel wavenumber.

to take into account when modelling the ULF wave propagation. Mariner 10 detected ULF waves with a peak frequency $f_{\rm obs} = 0.5\,$Hz during its first passage of Mercury's magnetosphere [6, 7]. The local proton gyrofrequency was $f_{\rm cH} = \omega_{\rm cH}/(2\pi) = 1.31\,$Hz. For proton majority plasmas with singly ionized heavy ions, $\mu = \mathcal{Z}_1/\mathcal{Z}_2 = A_2$, i.e. μ equals the atomic mass of heavy minority ions. If we assume $k_\parallel = 0$, the observed wave frequency leads to unrealistically high sodium concentrations. In fact, if we take the Buchsbaum frequency, i.e. the IIH frequency relevant for $k_\parallel = 0$, as that corresponding to the observed peak in the spectrogram, $\omega_{\rm S0}/\omega_{c2} = \sqrt{[1-(1-\mu)f_2]/[1-(1-1/\mu)f_2]} = 8.78$, then it leads to $f_2 = X[{\rm Na}^+] = 79.5\%$. However, if we assume the detection of waves with $k_\parallel \simeq k_\parallel^*$ and take the crossover frequency to interpret the observed wave, one concludes $X[{\rm Na}^+] = 14.4\%$. While the former estimate is unrealistically high, the latter is within the 10–50% range for $\rm Na^+$ identified earlier. The reason for such a large difference between the estimates of the sodium concentrations is because $\omega_{\rm S0}$ and $\omega_{\rm cr}$ include terms μf_2 and $\mu^2 f_2$, respectively. Since $\mu = A_{\rm Na^+} = 23 \gg 1$, a much lower sodium concentration is needed for the crossover frequency to match the observed event frequency.

The above result for the crossover frequency defined as $\epsilon_2(\omega_{\rm cr}) = 0$ can easily be generalized to treat multiple ion species. Using the notations introduced before, the normalized crossover frequency is given by the solution of the equation $\sum_i f_i/(\tilde{\omega}_{\rm cr}^2 - \mathcal{Z}_i^2) = 0$. In plasmas including only two ion species it equals to $\tilde{\omega}_{\rm cr0} = \sqrt{\mathcal{Z}_2^2 + (\mathcal{Z}_1^2 - \mathcal{Z}_2^2)f_2}$. Accounting for additional ion species (impurities), we have $\tilde{\omega}_{\rm cr} = \sqrt{\tilde{\omega}_{\rm cr0}^2 + \epsilon}$, and the effect of impurities on the crossover frequency is described by $\epsilon = \sum_{\rm imp} k_{12} f_2 f_{\rm imp} \times \left(1 + \sum_{\rm imp} k_{22} f_{\rm imp}\right)$, with $k_{12} = (\mathcal{Z}_1^2 - \mathcal{Z}_2^2)(\mathcal{Z}_1^2 - \mathcal{Z}_{\rm imp}^2)/(\tilde{\omega}_{\rm cr0}^2 - \mathcal{Z}_{\rm imp}^2)$, $k_{22} = (\mathcal{Z}_1^2 - \mathcal{Z}_{\rm imp}^2)(\mathcal{Z}_2^2 - \mathcal{Z}_{\rm imp}^2)/(\tilde{\omega}_{\rm cr0}^2 - \mathcal{Z}_{\rm imp}^2)^2$. Using these formulae, one can show that the ratio of the minority concentration satisfying the crossover condition in plasmas with and without impurities is given by

$$\frac{f_2^*}{f_2} = 1 - \sum_{\rm imp} \frac{\mathcal{Z}_1^2 - \mathcal{Z}_{\rm imp}^2}{\tilde{\omega}_{\rm cr0}^2 - \mathcal{Z}_{\rm imp}^2} f_{\rm imp} \approx 1 - \sum_{\rm imp} \frac{\mathcal{Z}_1^2 - \mathcal{Z}_{\rm imp}^2}{\mathcal{Z}_1^2 f_2 - \mathcal{Z}_{\rm imp}^2} f_{\rm imp} \,. \tag{9}$$

For the right hand side of Eq. (9) we made use of $\tilde{\omega}_{\rm cr0} \simeq \mathcal{Z}_1 f_2^{1/2}$ if $\mu \gg 1$.

In the case of Mercury and ignoring the presence of He^{2+} ions, the observed wave frequency corresponds to the sodium concentration 14.4%. According to Eq. (9), the presence of helium ions leads to an increase in the sodium concentration, $X[{\rm Na}^+] = 0.144 + 2.069\,X[{\rm He}^{2+}]$, in agreement with the formula given in Ref. [14]. For a plasma including 5% of helium ions, the sodium concentration increases up to 24.8%. Thus, using a two-ion species approximation only for the crossover frequency can lead to a significant underestimate of $X[{\rm Na}^+]$. If we assume that the ratio between the concentrations of helium ions and protons is the same as in the solar wind ($X[{\rm H}^+]/X[{\rm He}^{2+}] = 73.2/3.05 = 96/4$), we find $X[{\rm Na}^+] = 20.7\%$.

In Ref. [5], a strong linearly polarized ULF emission peaking at $f_{\rm obs} = 1\,{\rm Hz}$ was observed in the magnetosphere of the Earth. The local helium cyclotron frequency was $f_{\rm c,He^+} \approx 0.57\,{\rm Hz}$. Then, from the ratio $f_{\rm obs}/f_{\rm cH} \approx 0.44$, one would conclude $X[{\rm He}^+] = 38.9\%$ assuming the Buchsbaum IIH frequency (corresponding to the assumption of $k_\parallel \simeq 0$). However, if the observed frequency instead is matched to the crossover frequency, one finds $X[{\rm He}^+] = 13.8\%$. The latter is in good agreement with the value used in simulations in Ref. [15]. For the Earth's magnetosphere the third ion species of importance is O^+, and it influences the crossover frequency such that $X[{\rm He}^+] = 0.138 - 0.731X[{\rm O}^+]$. In contrast to Mercury's case, accounting for the additional impurity species leads to a reduction of the heavy ion concentration responsible for the crossover frequency. If the oxygen concentration is 3%, then the helium concentrations drops to 11.6%.

Conclusions Mode conversion of the fast magnetosonic waves at the IIH resonance was suggested earlier for the interpretation of linearly polarized waves detected by satellites. However, in this paper we have shown that the Buchsbaum IIH resonance frequency relevant for the waves with a small parallel wavenumber is inconsistent with experimental observations in Earth's and Mercury's magnetospheres due to negligible wave tunneling at $k_{\|} = 0$ for typical parameters of such plasmas. Instead, we show that tunneling and conversion efficiency may become significant for waves excited with parallel wavenumbers close to a critical value $k_{\|}^{*}$, corresponding to condition for which the MC layer width approaches zero, allowing thus the detection of ULF events. Under such conditions, the measured MC frequency is close to the crossover frequency (but somewhat below), $\omega = \omega_{\rm S} \simeq \omega_{\rm cr}$ rather than to the Buchsbaum IIH frequency, $\omega_{\rm S0}$. The analytical formulae for the critical parallel wavenumber and the crossover frequency in the presence of multiple ion species can be used for making estimates of the heavy ion concentrations in various planetary magnetospheres.

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[1] D Van Eester et al, Plasma Phys Control Fusion **54**, 074009 (2012).

[2] D-H Lee et al, J Geophys Res **113**, A11212 (2008).

[3] M André, J Plasma Phys **33**, 1 (1985).

[4] A Roux et al, J Geophys Res **87**, 8174 (1982).

[5] S Perraut et al, J Geophys Res **89**, 195 (1984).

[6] C T Russell, J Geophys Res **16**, 1253 (1989).

[7] E-H Kim et al, J Geophys Res **113**, A11207 (2008).

[8] S B Korsholm et al, Phys Rev Lett **106**, 165004 (2011).

[9] W W Heidbrink, G W Watson and K H Burrell, Rev Sci Instrum **75** 3862 (2004).

[10] T H Stix 1992 *Waves in Plasmas* (New York: AIP).

[11] Ye O Kazakov et al, Nucl Fusion **53**, 053014 (2013).

[12] D G Swanson, *Theory of Mode Conversion and Tunneling in Inhomogeneous Plasmas*, Wiley (1998).

[13] K G Budden, *The Propagation of Radio Waves*, Cambride University Press (1985).

[14] C Othmer et al, J Geophys Res **104**, 369 (1999).

[15] Y Hu et al, J Geophys Res **115**, A09218 (2010).