Neoclassical Radial Impurity Flux in a Mixed Collisonality Stellarator Plasma

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Abstract: Neoclassical impurity accumulation must be controlled in stellarators. We calculate analytically the radial flux of a heavy impurity in a mixed collisionality plasma with the bulk in the $1/\nu$ regime. We evaluate the effect of the impurity on the bootstrap current and identified temperature screening of impurity flux.

1. Overview
- Neoclassical impurity accumulation must be controlled in stellarators.
- We calculate analytically the radial flux of a heavy impurity in a mixed collisionality plasma with the bulk in the $1/\nu$ regime.
- We evaluate the effect of the impurity on the bootstrap current.
- Identified temperature screening of impurity flux.

2. Formulation
- Radial impurity flux can be obtained from a flux-friction relation [1]

$$\langle \mathbf{v} \mathbf{v} \rangle = \frac{1}{Z_T} \left[ u B R_{ci} + \left( p_z - p_j \right) \frac{\nabla T_B}{2B} \right]$$

- the equilibrium current function $u = j_0 / \mu B$
- collisional system treated in [1], relevant for edge plasma.
- Here treat mixed collisionality system
- single heavy, highly charged, collisional impurity species $z$
- low collisionality background hydrogenic bulk $\nu$, in $1/\nu$ regime.
- Neglect pressure anisotropy when collisionality $\nu_i$ satisfies

$$\nu_i \ll Z_i \frac{\langle n_i \rangle}{m_i \langle Z_i \rangle}$$

- to give finite friction drive, bulk impurity collisionality cannot be too low.
- Evaluate friction via bulk ion distribution $R_{ji} = R_{ji} = -R_{ji}$.
- neglect effect of radial electric field $E_r$ on particle orbits.
- retain $E_r$ as driving force at large aspect ratio [2].
- Model collision operators [3] used to evaluate bulk ion distribution.
- disparate mass simplifies bulk-impurity collisions

$$R_{ji} = n_i \int \frac{\nu_j}{\nu_i} f_j(v_j) f_i(v_i) - m_i n_i \nu_i \langle v_i \rangle$$

- impurity flow from continuity

$$\langle BV_j \rangle = \frac{1}{n_i n_j} \frac{\partial p_{ji}}{\partial v_j} + \frac{\partial n_i}{\partial v_i} \langle u B \rangle + \frac{K_i(v_0)}{n_i} \langle Z_i \rangle$$

- parallel force balance sets $K_i(v_0)$, requires finite bulk ion density for consistency

$$\nu_i \ll Z_i \frac{\langle n_i \rangle}{m_i \langle Z_i \rangle} < 1$$

- model bulk self-collision operator includes momentum restoring term $\propto \mathbf{V}_i$.
- affects piece of distribution odd in $\mathbf{v}_i$, giving flow and friction.
- $\mathbf{V}_i$ introduces velocity space average $v_j / n_j \approx 1.17$
- introduced level of impurity content via $\zeta = \frac{\nu_i}{\nu} = \frac{Z_i}{Z} - 1 + \frac{\nu_i}{\nu}$
- to evaluate bootstrap current with impurities approximate energy dependence of bulk ion self-collision frequency $\nu_i \approx \nu_i / \zeta$

- Effect of geometry appears via functions
- function describing effect of trapping, where $\lambda = v_j / v_B$

$$f_i(y) = \langle B^2 \rangle \frac{1}{\sqrt{2\pi}} \int \frac{\partial \phi}{\partial \lambda} \frac{\langle y \rangle}{\Delta B} d\phi$$

- integrated contributions from the particle drifts appear, as in [4]

$$\langle g_i \rangle = \frac{1}{\sqrt{2\pi}} \int \frac{\langle y \rangle}{\Delta B} d\phi \approx \frac{1}{\langle B \rangle}$$

- the integration is along a field line from arbitrary starting location $t_0$.

- Define usual radial driving gradients

$$A_{\nu_j} = \frac{d \ln p_{ji}}{d \phi} + Z_i e B_{ji} \frac{d \phi}{d \phi}$$

3. Radial Impurity Flux
- Impurity flux can be evaluated using calculated distribution

$$\langle \mathbf{f} \mathbf{v} \rangle = \frac{m_i p_0}{Z_T e^{2} T_j} \left[ \langle A_i \rangle^2 + \langle A_i \rangle^2 + \langle A_i \rangle^2 \right]$$

- giving transport coefficients

$$\langle \mathbf{L} \rangle = -\frac{1}{Z_T} \left( \langle u B \rangle - \langle u B \rangle \right)$$

- are independent of impurity content.
- compare to high collisionality relation, $\langle \mathbf{L} \rangle = -\mathbf{L}_{\parallel} / Z_i$.
- $\mathbf{L}_{\parallel}$ negative definite by Schwartz inequality, required by entropy production
- $\mathbf{L}_{\parallel}$ positive and gives temperature screening with $\mathbf{L}_{\parallel}$
- independent of equilibrium geometry
- taking tokamak limit reproduces usual temperature screening.
- Large impurity charge, total direct drive by radial electric field $\propto Z_i \mathbf{L}_{\parallel} + \mathbf{L}_{\parallel} = 0$

4. Bootstrap Current
- Bulk ion contribution to bootstrap current evaluated directly from distribution function

$$\langle f_i \rangle = p_0 \left[ \langle A_i \rangle^2 + \langle A_i \rangle^2 \right]$$

- giving coefficients

$$\mathbf{L}_{\parallel} = -\frac{1}{\langle u \rangle} \left( \langle u B \rangle - \langle u B \rangle \right)$$

- no direct dependence on impurity gradients.
- dependent on impurity content, $\zeta$.
- reduces to existing results obtained in pure plasma limit [4].
- low collisionality trapping effects appear as geometry factor $g_i$
- retaining only pitch angle scattering bulk ion collision operator gives

$$\langle f_i \rangle \approx \frac{1}{\langle u \rangle} \left( \zeta - 1 \right) \frac{1}{\langle u \rangle}$$

- tokamak limit [3]. $f_i (1) \rightarrow f_i (g_i) + \langle u B \rangle$.

5. Underway: Comparison with SFINCS
- SFINCS can be used to evaluate the transport coefficients and bootstrap current.
- continuum, 4D drift-kinetic equation solver, full linearised collision operator.
- treating W7-X equilibrium, collisionality constraints set range of applicability.
- temperature screening effect already indicated in [5].

6. Summary
- Analytic forms determined for radial impurity flux and bootstrap current in mixed collisionality stellarator plasma.
- Temperature screening of impurity flux arises.
- Impurity flux and bootstrap current may be optimised individually.
- Step towards treating effect of finite $E_r$, analytically, bulk ions will move into $\sqrt{B}$ regime.


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