Neoclassical Radial Impurity Flux in a Mixed Collisionality Stellarator Plasma

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Overview

- **Neoclassical impurity accumulation must be controlled in stellarators**
- **We calculate analytically the radial flux of a heavy impurity in a mixed collisionality plasma with the bulk in the $1/\nu$ regime**
- **We evaluate the effect of the impurity on the bootstrap current**
- **Identified temperature screening of impurity flux**

Formulation

• Radial impurity flux can be obtained from a **flux-friction relation** [1]

$$\langle \mathbf{\Gamma}_z \cdot \nabla\psi \rangle = \frac{1}{Z_z e}\left\langle uBR_{z\|} + \left(p_{z\|} - p_{z\perp}\right)\frac{\nabla_\|(uB^2)}{2B}\right\rangle$$

– the equilibrium current function $u = j_{0\|}/p_0' B$
– collisional system treated in [1], relevant for edge plasma

• Here treat **mixed collisionality** system
– **single heavy, highly charged, collisional impurity** species z
– low collisionality background **hydrogenic bulk i in $1/\nu$ regime**

• Neglect pressure anisotropy drive when **collisionality ν_* satisfies**

$$\frac{1}{\nu_{*iz}} \ll \frac{n_i Z_z}{n_z}\sqrt{\frac{m_i}{m_z}}\nu_{*zz}$$

– to give finite friction drive, bulk ion collisionality cannot be too low

• **Evaluate friction** via bulk ion distribution $R_{z\|} = R_{zi\|} = -R_{iz\|}$
– neglect effect of radial electric field E_r on particle orbits
 - retain E_r as driving force at large aspect ratio [2]
– **model collision operators** [3] used to evaluate bulk ion distribution
 - disparate mass simplifies bulk-impurity collisions

$$R_{zi\|} = m_i \int \nu_D^{iz}(v) v_\| f_{i1} d^3v - \frac{m_i n_{i0}}{\tau_{iz}} V_{z\|}$$

 - impurity flow from continuity

$$\langle BV_{z\|}\rangle = \left(\frac{1}{n_{z0} Z_z e}\frac{dp_{z0}}{d\psi} + \frac{d\phi_0}{d\psi}\right)\langle uB^2\rangle + \frac{K_z(\psi)}{n_{z0}}\langle B^2\rangle$$

 - parallel force balance sets $K(\psi)$, requires **finite bulk ion density for consistency**

$$\nu_{*iz}\frac{n_z Z_z}{n_i Z_i^2} < 1$$

 - model bulk self-collision operator includes momentum restoring term $\propto \mathcal{V}_\|$
  - **affects piece of distribution odd in $v_\|$**, giving flow and friction
  - evaluating $\mathcal{V}_\|$ introduces velocity space averages $\eta_2/\eta_1 \approx 1.17$
– introduced level of impurity content via $\zeta = \frac{n_z Z_z^2}{n_i Z_i^2} = \frac{Z_{\rm eff}}{Z_i} - 1 + Z_{\rm eff}\frac{n_z Z_z}{n_i Z_i^2}$
 - to evaluate bootstrap current with impurities approximate energy dependence of bulk ion self-collision frequency $\nu_D^{ii} \approx \nu_D^{iz}\tau_{iz}/\tau_{ii}$

• Effect of **geometry appears via** functions
– function describing effect of trapping, where $\lambda = v_\perp^2/v^2 B$

$$f_s(y) = \langle B^2\rangle \frac{3}{4}\int_0^{1/B_{max}} \frac{\langle y\rangle}{\langle\sqrt{1-\lambda B}\rangle}\lambda d\lambda$$

– integrated contributions from the particle drifts appear, as in [4]

$$\langle g_2\rangle = \left\langle B^2 \int_{l_0}^{l}\left(\hat{\mathbf{b}}\times\nabla\psi\right)\cdot\nabla\left(\frac{1}{B^2}\right)dl'\right\rangle \equiv -\langle \mathbf{uB^2}\rangle$$

$$\langle g_4\rangle = \left\langle \sqrt{1-\lambda B}\int_{l_0}^{l}\left(\hat{\mathbf{b}}\times\nabla\psi\right)\cdot\nabla\left(\frac{1}{\sqrt{1-\lambda B}}\right)dl'\right\rangle$$

- the integration is along a field line from arbitrary starting location l_0

• Define usual **radial driving gradients**

$$A_{1a} = \frac{d\ln p_{a0}}{d\psi} + \frac{Z_a e}{T_{a0}}\frac{d\phi_0}{d\psi} \qquad A_{2a} = \frac{d\ln T_{a0}}{d\psi}$$

Radial Impurity Flux

• Impurity flux can be evaluated using calculated distribution

$$\langle \Gamma_z \cdot \nabla\psi\rangle = \frac{m_i p_{i0}}{Z_z Z_i e^2 \tau_{iz}}\left[\mathcal{L}_{11}^{zz}A_{1z} + \mathcal{L}_{11}^{zi}A_{1i} + \mathcal{L}_{12}^{zi}A_{2i}\right]$$

giving **transport coefficients**

$$\mathcal{L}_{11}^{zz} = -\frac{1}{Z_z}\left[\langle u^2B^2\rangle - \frac{\langle uB^2\rangle^2}{\langle B^2\rangle}\right]$$

$$\mathcal{L}_{11}^{zi} = -\left[\langle ug_2\rangle - \frac{\langle uB^2\rangle}{\langle B^2\rangle}\langle g_2\rangle\right] = -Z_z\mathcal{L}_{11}^{zz}$$

$$\mathcal{L}_{12}^{zi} = -\frac{3}{2}\mathcal{L}_{11}^{zi}$$

– are independent of impurity content
– compare to high collisionality relation, $\mathcal{L}_{11}^{zz} = -\mathcal{L}_{11}^{zi}/Z_z$

• $\mathcal{L}_{11}^{zz}$ negative definite by Schwartz inequality, required by entropy production
$\Rightarrow$ $\mathcal{L}_{11}^{zi}$ positive and gives temperature screening with $\mathcal{L}_{12}^{zi}$
– independent of equilibrium geometry
– taking tokamak limit reproduces usual temperature screening

• Large impurity charge, total direct drive by radial electric field $\propto Z_z\mathcal{L}_{11}^{zz} + \mathcal{L}_{11}^{zi} = 0$

Bootstrap Current

• Bulk ion contribution to bootstrap current evaluated directly from distribution function

$$\left\langle J_\|^i B\right\rangle = p_{i0}\left[\mathcal{L}_{31}^{zi}A_{1i} + \mathcal{L}_{32}^{zi}A_{2i}\right]$$

giving **coefficients**

$$\mathcal{L}_{31}^{zi} = \frac{1}{1-f_s(1)}\left[f_s(g_4) + \langle uB^2\rangle\right]$$

$$\mathcal{L}_{32}^{zi} = -\frac{f_s(1)}{1+\zeta}\left(\zeta + \frac{2\eta_2}{3\eta_1}\right)\frac{3}{2}\mathcal{L}_{31}^{zi}$$

– no direct dependence on impurity gradients
– dependent on impurity content, ζ
– reduces to existing results obtained in pure plasma limit [4]
– low collisionality trapping effects appear as geometry factor g_4
– retaining only pitch angle scattering bulk ion collision operator gives

$$\left\langle J_\|^i B\right\rangle^{PAS} = p_{i0}\left[f_s(g_4) + \langle uB^2\rangle\right]A_{1i}$$

– tokamak limit [3], $f_s(1) \to f_c$, $f_s(g_4) + \langle uB^2\rangle \to -f_t I(\psi)$

Underway: Comparison with SFINCS

• SFINCS can be used to evaluate the transport coefficients and bootstrap current
– continuum, 4D drift-kinetic equation solver, full linearised collision operator
– treating **W7-X equilibrium**, collisionality constraints set range of applicability
– temperature screening effect already indicated in [5]

Summary

- **Analytic forms determined for radial impurity flux and bootstrap current in mixed collisionality stellarator plasma**
- **Temperature screening of impurity flux arises**
- **Impurity flux and bootstrap current may be optimised individually**
- **Step towards treating effect of finite E_r analytically, bulk ions will move into $\sqrt{\nu}$ regime**

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