Effect of neutral atoms on tokamak edge plasmas

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Neutral atoms can significantly influence the physics of tokamak edge plasmas, e.g., by affecting the radial electric field and plasma flow there, which may, in turn, be important for plasma confinement. Earlier work [Fülöp *et al.*, Phys. Plasmas **5**, 3969 (1998)], assuming *short mean-free path* neutrals and Pfirsch–Schlüter ions, has shown that the ion-neutral coupling through charge-exchange affects the neoclassical flow velocity significantly. However, the mean-free path of the neutrals is not always small in comparison with the radial scale length of densities and temperatures in the edge pedestal. It is therefore desirable to determine what happens in the limit when the neutral mean-free path is comparable with the scale length. In the present work a self-similar solution for the neutral distribution function allowing for strong temperature and density variation is used, following the analysis of Helander and Krasheninnikov [Phys. Plasmas **3**, 226 (1995)]. The self-similar solution is possible if the ratio of the mean-free path to the temperature and density scale length is constant throughout the edge plasma. The resulting neutral distribution function is used to investigate the neutral effects on the ion flow and electrostatic potential as this ratio varies from much less than one to order unity.
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I. BACKGROUND

Neutral atoms are abundant in the tokamak edge plasma. Through charge-exchange (CX) processes, they can modify the ion flow in the edge plasma. Even for rather small neutral to plasma density ratios, the large diffusivity of the neutrals in combination with the ion-neutral coupling can directly modify the ion distribution function in this region and thereby alter the parallel momentum constraint that determines the parallel ion flow. In addition, there is a large neutral flux of toroidal angular momentum that can modify or even determine the radial electric field at the edge.

The effect of neutrals can be important for the global confinement of the plasma, since the sharp radial and poloidal variation of the neutrals can introduce a strong shear into the poloidal ion flow and thereby the electrostatic potential. Consequently, the neutrals may influence or even be responsible for the region of sheared flow observed at the edge in reduced transport regimes.[1–3]

Earlier work assuming *short mean-free path* neutrals and plateau[4] or Pfirsch-Schlüter,[5,6] ions has showed that the neutral diffusion affects the neoclassical flow velocity significantly. However, the mean-free path of the neutrals is not always small in comparison with the radial scale length of densities and temperatures in the edge pedestal. It is therefore interesting to investigate the limit when the neutral mean-free path is comparable with the scale length.

In Ref. 7 the neutral kinetic equation has been solved allowing for strong temperature variation. The solution is possible if the ratio of the mean-free path to the temperature and density scale length is constant throughout the edge plasma, in which case it is possible to introduce self-similar variables that reduce the dimensionality of the problem.

The aims of the treatment presented here are to use the self-similar neutral distribution function from Ref. 7 to investigate neutral effects on the ion flow and the electrostatic potential and to compare the results with those found in the short mean-free path limit.

The structure of this article is as follows. In Sec. II we present a self-similar solution of the neutral kinetic equation and discuss the choice of free parameters appearing in the solution. In Sec. III we investigate the neutral effects on the parallel momentum equation that determines the parallel ion flow and we compare the results with the short mean-free path regime. We also evaluate the neutral contribution to the parallel ion flow numerically. In Sec. IV we calculate the neutral viscosity, and discuss the effect of the neutral atoms on the electrostatic potential. Finally in Sec. V we summarize our conclusions.

II. SELF-SIMILAR VARIABLES IN A SIMPLE CX MODEL

The Boltzmann equation for neutral atoms in steady state is

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$$v_r \nabla_r f_n = X(f_n, f_i) - K_z n_i f_n(\mathbf{v}), \tag{1}$$

where f_n and f_i are the neutral and ion distribution functions, v_r denotes the component of the velocity *in the inward direction of the gradients* of the background plasma, the CX operator is given by

$$X(f_n, f_i) = \int \sigma(|\mathbf{v}-\mathbf{v}'|)|\mathbf{v}-\mathbf{v}'|[f_i(\mathbf{v}) f_n(\mathbf{v}') - f_n(\mathbf{v}) f_i(\mathbf{v}')] d^3v', \tag{2}$$

and K_z is the ionization rate constant.

If the CX cross section is assumed to be inversely proportional to $|\mathbf{v}-\mathbf{v}'|$, then the CX operator is simplified as $\sigma(|\mathbf{v}-\mathbf{v}'|)|\mathbf{v}-\mathbf{v}'| = K_x$ is a constant. Then Eq. (1) leads to

$$v_r \frac{\partial f_n}{\partial y} + f_n = \frac{n_n \beta}{n_i} f_i, \tag{3}$$

where $\partial/\partial y \equiv [n_i(K_x+K_z)]^{-1}\nabla_r$ and $\beta \equiv K_x/(K_x+K_z) \leq 1$.

We assume that the ratio of mean-free path to the macroscopic scale length is constant throughout the region of interest:

$$\gamma \equiv v_T \frac{d \ln T}{dy} = \frac{v_T}{n_i(K_x+K_z)} \nabla_r \ln T = \text{const}, \tag{4}$$

where $v_T^2 = 2T/M$. Reference 7 showed that a self-similar solution to Eq. (3) exists in the form

$$f_n(y,\mathbf{v}) = \frac{N}{T^\alpha(y)} F(\mathbf{u}), \tag{5}$$

with α a free parameter, $\mathbf{u} = \mathbf{v}/v_T$, N is a constant, F is normalized such that

$$\int F(\mathbf{u})\, d^3u = 1, \tag{6}$$

and

$$F(\mathbf{u}) = \begin{cases} \dfrac{2\beta}{\gamma u_r} \displaystyle\int_0^1 \hat{f}\left(\frac{\mathbf{u}}{s}\right) \exp\left(\frac{2(s-1)}{\gamma u_r}\right) \frac{ds}{s^{2\alpha}}, & u_r > 0, \\ -\dfrac{2\beta}{\gamma u_r} \displaystyle\int_1^\infty \hat{f}\left(\frac{\mathbf{u}}{s}\right) \exp\left(\frac{2(s-1)}{\gamma u_r}\right) \frac{ds}{s^{2\alpha}}, & u_r < 0, \end{cases} \tag{7}$$

where $\hat{f} \equiv f_i v_T^3/n_i$. Note that it is assumed that $\gamma > 0$, so that the temperature increases with y. The restriction that $\gamma = \text{const}$ couples the density and temperature profiles through Eq. (4), but one of these profiles is still arbitrary.

The parameters α, β and γ are not independent since Eq. (6) imposes a constraint on the solution. If the ion distribution is Maxwellian, $\hat{f}(\mathbf{u}) = \exp(-u^2)/\pi^{3/2}$, so that T represents the ion temperature, Eq. (6) leads to the constraint

$$\frac{1}{\beta} = \frac{2}{\gamma \pi^{1/2}} \int_0^\infty \frac{du_r}{u_r} \int_0^\infty \exp\left(-\frac{2|s-1|}{\gamma u_r} - \frac{u_r^2}{s^2}\right) \frac{ds}{s^{2(\alpha-1)}} \equiv P(\alpha,\gamma). \tag{8}$$

Equation (8) determines one of the three parameters as a function of the two other parameters. For a simpler way of estimating the dependence of the three different parameters, it is useful to take the limit of short mean-free path of Eq. (8):

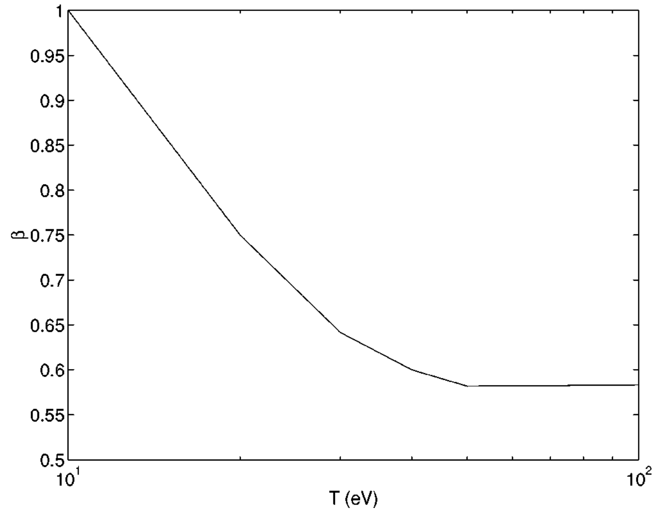

FIG. 1. β as a function of temperature.

$$P(\alpha,\gamma) = 1 + (\alpha-2)(\alpha-5/2)\gamma^2/2, \gamma \to 0. \tag{9}$$

In the short mean-free path limit for $\beta = 1$ (neglecting the ionization), the roots of Eq. (8) are $\alpha = 2$ and $\alpha = 2.5$. The parameter β depends on the relative importance of CX and ionization, and it ranges from about 0.5 to 1 for typical edge plasma temperatures. Figure 1 shows β as a function of temperature. For $\beta < 1$ and $\gamma > 0$, one of the roots is less than 2 and can be discarded as unphysical, since only roots $\alpha > 3$ will turn out to be of interest. The other root is higher than 3, if γ is not too large (see Fig. 2). For fixed α, the value of β decreases with γ, as shown in Fig. 3.

Forming the neutral density $n_n = \int d^3v f_n$ gives

$$n_n = N(2/M)^{3/2}/T^{(\alpha-3/2)} = N \frac{v_T^3}{T^\alpha}.$$

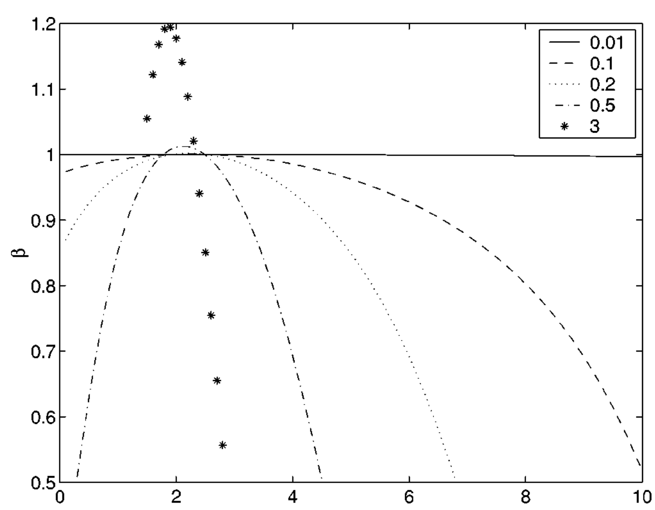

FIG. 2. β as a function of α for different γ (see inset) from Eq. (8). For a given value of $\beta < 1$, the two roots can be determined for each value of γ from Fig. 1, but only roots with $\alpha > 3$ are of physical interest.

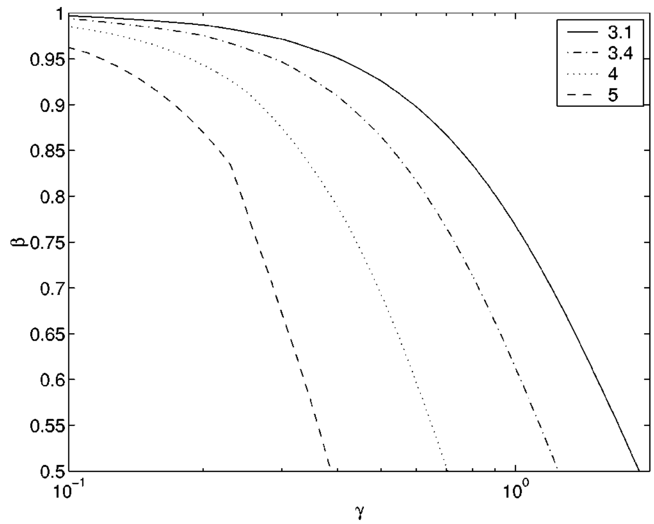

FIG. 3. β as a function of γ for different α (see inset), calculated from Eq. (8).

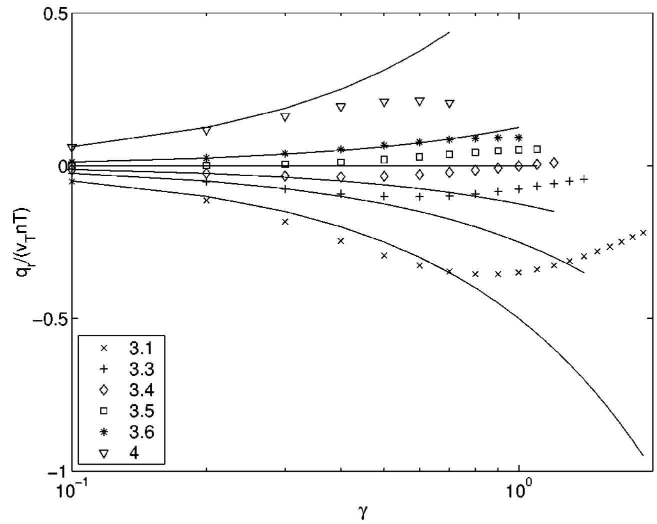

FIG. 4. Neutral heat flux as a function of γ for different α (see inset).

Consequently, values of $\alpha > 1.5$ correspond to a realistic situation when the neutral density decreases going into the plasma, while the temperature increases. Note that the parameter α cannot be arbitrarily large since it is proportional to the ratio of the neutral density and temperature scale lengths:

$$\alpha - 3/2 = -\frac{d \ln n_n}{dr} \Bigg/ \frac{d \ln T}{dr}.$$

Furthermore, the particle flux associated with the neutral distribution function,

$$j_r = \int f_n v_r d^3v = \frac{(1-\beta) n_n v_T}{\gamma(\alpha-2)}, \tag{10}$$

should be positive (inward) since the neutrals are moving into the plasma, which requires that $\alpha > 2$.[7] Another restriction on the value of α is found by inspecting the neutral heat flux

$$q_r = \int f_n \frac{Mv^2}{2} v_r d^3v, \tag{11}$$

which diverges for $\alpha \leq 3$,[7] and can be both inward or outward, depending on the relative frequency of the charge-exchange and ionization processes. In the short mean-free path limit, for Maxwellian ions the neutral heat flux is

$$\lim_{\gamma \to 0} q_r = \gamma \frac{5(\alpha - 3.5)}{4} v_T n T. \tag{12}$$

Note that $3 < \alpha < 3.5$ corresponds to the case when neutrals will carry heat out of the plasma (after undergoing multiple charge-exchange events), while $\alpha > 3.5$ corresponds to an inward neutral heat flux. Figure 4 shows the neutral heat flux as a function of γ for different α. For physical solutions, $\alpha > 3$ can be fixed and then β will be determined by Eq. (8) for a range of different values of γ.

The neutral temperature can be calculated as

$$\begin{aligned} T_n &= \frac{2}{3n_n} \int f_n \frac{Mv^2}{2} d^3v \\ &= \frac{16\beta T}{3\pi^{3/2}\gamma} \int_0^\infty u^3 du \int_0^{\pi/2} dt \int_0^1 \frac{ds}{s^{2\alpha}} \\ &\quad \times \left(e^{-u^2/s^2} K_0\left(\frac{2(1-s)}{\gamma u \cos t} \right) \right. \\ &\quad \left. + e^{-u^2 s^2} K_0\left(\frac{2(1-s)}{\gamma s u \cos t} \right) s^{4\alpha-2} \right), \end{aligned} \tag{13}$$

which in the limit of short mean-free path reduces to

$$\begin{aligned} \lim_{\gamma \to 0} T_n &= \beta T \left(1 + \frac{5}{6}(\alpha-3)\left(\alpha - \frac{7}{2}\right)\gamma^2 \right) \\ &= T \frac{1 + (\frac{5}{6})(\alpha-3)(\alpha-\frac{7}{2})\gamma^2}{1 + (\frac{1}{2})(\alpha-2)(\alpha-\frac{5}{2})\gamma^2}. \end{aligned}$$

Note that ionization is necessary to keep $T_n < T$ for $\gamma \to 0$, and that finite γ enhances the departure. For $\alpha > 7$, the neutral temperature can exceed the ion temperature since neutrals traveling outwards after undergoing CX events can be considerably hotter than those of the local background. Figure 5 shows the ratio T_n/T calculated from (13) as a function of γ for different α.

If $\gamma\alpha$ is large, the value of β obtained from Eq. (8) is too small to be physically interesting. Low values of β would mean that the ionization dominates over CX, which is not true for typical edge parameters (recall Fig. 1). Estimates from the numerical calculations indicate that only $\gamma \lesssim 6/\alpha - 1$ are of interest for $1 \geq \beta > 0.5$.

Thus, the physical requirements of finite neutral heat flux and $\beta > 0.5$ limit our choices of α and γ to $\alpha > 3$ and $\gamma \lesssim 2$. In this region, we will show that there are quantitative differences between the self-similar and short mean-free path results for the neutral modification to the ion flow, the electrostatic potential, and the departure of the neutral temperature from that of the ions, but that these differences are less

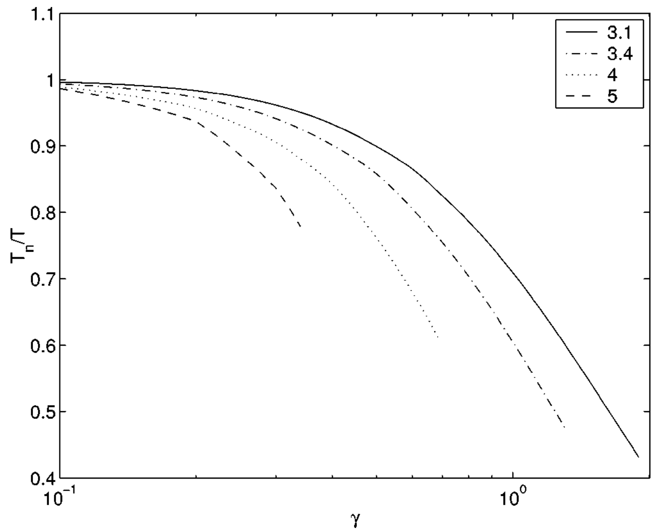

FIG. 5. The ratio of the neutral to ion temperatures as a function of γ for different α (see inset).

than a factor of 2. However, as $\alpha \rightarrow 3$, the short mean-free path approximation for the neutral heat flux fails at smaller γ than in cases with higher α.

III. NEUTRAL EFFECTS ON THE ION FLOW

We begin with the ion drift-kinetic equation and assume the neutral distribution function is known from Eqs. (5)–(7):

$$(v_{\|}\hat{n}+\mathbf{v}_d)\cdot\nabla f_i - C_{ii}(f_i) = -\langle X(f_i,f_n)\rangle_{\varphi} = -K_x(n_n f_i - n_i f_n). \tag{14}$$

The ion flow velocity can be determined from the parallel momentum constraint equation obtained by taking the $M\mathbf{v}\cdot\mathbf{B}=MBv_{\|}$ moment of the sum of ion, neutral and the electron kinetic equations, and performing a flux-surface average. This equation is[5]

$$\langle (p_{\|}-p_{\perp})\nabla_{\|}B\rangle = 0, \tag{15}$$

where $\langle\cdots\rangle$ denotes flux-surface average, and we have defined

$$p_{\|}=\sum_s m_s\int d^3v v_{\|}^2 f_s \quad\text{and}\quad p_{\perp}=\sum_s m_s\int d^3v\,\frac{v_{\perp}^2}{2}f_s,$$

where the subscript s denotes different species. Only the ions and neutrals contribute, since the electrons are isotropic in velocity space to the requisite order.

A. Modification of the ion distribution function

The CX term modifies the ion distribution function as in Refs. 4 and 5 so we may write $f_i=f_{\text{Hazeltine}}+h$, where $f_{\text{Hazeltine}}$ is the Pfirsch–Schlüter solution without the CX term, as given in Ref. 8, and $h=\Sigma_j H_j(v)P_j(\xi)=\Sigma_j h_j$ with $\xi=v_{\|}/v$ and P_j is the Legendre polynomial. To calculate the modification of the ion flow, we need only calculate h_2 since $p_{\|}-p_{\perp}=M\int d^3v(v_{\|}^2-v_{\perp}^2/2)f_i=M\int d^3v v^2P_2(\xi)f_i$ where $P_2(\xi)=(3\xi^2-1)/2$.

For ions in the Pfirsch–Schlüter regime, the mean-free path is shorter than the connection length so to lowest order we need only solve

$$C_{ii}(h)=K_x(n_n f_i - n_i f_n) \tag{16}$$

for h_2. As in all neoclassical transport theory, the ions are assumed to be close to local thermodynamic equilbrium, so that f_i may be replaced by a Maxwellian on the right of Eq. (16). Expanding the right side in Legendre polynomials using

$$-K_x(n_n f_{Mi}-n_i f_n)=\sum_j P_j(\xi)V_j(v) \tag{17}$$

gives

$$n_iK_x\int_{-1}^{1}d\xi P_2(\xi)f_n = V_2(v)\int_{-1}^{1}d\xi P_2^2(\xi)=\frac{2}{5}V_2(v), \tag{18}$$

since $\int_{-1}^{1}d\xi P_j(\xi)P_k(\xi)=0$ for $j\neq k$ and $\int_{-1}^{1}d\xi(P_2(\xi))^2=2/5$. Therefore, we need only solve

$$C_{ii}(h_2)=-\frac{5}{2}P_2(\xi)n_iK_x\int_{-1}^{1}d\xi P_2(\xi)f_n. \tag{19}$$

To solve for h_2, we let $h_2=gf_M$ and $\beta_2=-P_2(\xi)V_2(v)$, then

$$C_l(gf_M)=\beta_2 \tag{20}$$

with C_l the linearized self-adjoint ion-ion collision operator. Equation (20) can be solved with a variational approach. We consider the functional $\Lambda=\int d^3v\hat{g}C_l(\hat{g}f_M)-2\int d^3v\hat{g}\beta_2$, which is variational ($\delta\Lambda=0$ for $\hat{g}=g$) and maximal ($\delta^2\Lambda<0$). We assume the simple trial function $\hat{g}=cv^2P_2(\xi)$ and find $\int d^3v\hat{h}C_l(\hat{g}f_M)=-(18p_iTc^2)/(5M^2\tau_i)$ and

$$\int d^3v\hat{g}\beta_2=-cn_iK_x\int d^3v v^2P_2(\xi)f_n$$

with $\tau_i=3T^{3/2}M^{1/2}/(4\sqrt{\pi}e^4n_i\ln\Lambda)$ the ion-ion collision time. The constant c can be determined from $\partial\Lambda/\partial c=0$, and as a result, using $h_2=\hat{g}f_M$ we obtain

$$h_2=\frac{5M^2\tau_iK_x}{18T^2}v^2f_MP_2(\xi)\int d^3v v^2P_2(\xi)f_n(\xi,v). \tag{21}$$

Evaluating $p_{\|}-p_{\perp}$ due to h_2 gives

$$(p_{\|}-p_{\perp})_{h_2}\equiv M\int d^3v v^2P_2(\xi)h_2 = \frac{5Mn_i\tau_iK_x}{6}\int d^3v v^2P_2(\xi)f_n \tag{22}$$

and the part due to f_n gives

$$(p_{\|}-p_{\perp})_n\equiv M\int d^3v v^2P_2(\xi)f_n, \tag{23}$$

which is smaller than the part due to h_2 by a factor $\tau_in_iK_x$. In other words, the direct neutral modification of the parallel momentum constraint is negligible compared with the effect of the neutral distortion of the ion distribution function.

Inserting Eq. (22) into Eq. (15) we obtain the parallel ion flow

$$V_{\parallel i}=V_{\parallel i}^{\mathrm{neo}}+V_n, \tag{24}$$

where we define the usual Pfirsch–Schlüter parallel flow velocity as[8]

$$V_{\parallel i}^{\mathrm{neo}}=-\frac{IT}{M\Omega}\left\{\frac{1}{p_i}\frac{\partial p_i}{\partial\psi}+\frac{Ze}{T}\frac{\partial\Phi}{\partial\psi}+\frac{1}{T}\frac{\partial T}{\partial\psi}\right.$$
$$\left.\times\left[1.8\frac{B^2}{\langle B^2\rangle}+0.05\frac{B^2\langle(\nabla_\parallel \ln B)^2\rangle}{\langle(\nabla_\parallel B)^2\rangle}\right]\right\},$$

where Ω is the ion gyrofrequency, $p_i=n_iT$, ψ is the poloidal flux function, $I=RB_t$ and B_t is the toroidal field. The contribution of the neutrals is given by

$$\langle(p_\parallel-p_\perp)_{h_2}\nabla_\parallel B\rangle\equiv\left\langle M\int d^3v\,v^2P_2(\xi)h_2\nabla_\parallel B\right\rangle$$
$$=\frac{5M\lambda_i\langle I_n\nabla_\parallel B\rangle}{6\lambda_x}\equiv 9p_i\tau_i\langle(\nabla_\parallel B)^2\rangle\frac{V_n}{B}, \tag{25}$$

where

$$I_n=\int d^3v\,v^2P_2(\xi)f_n$$
$$=\int_0^\infty dv\int_{-1}^1 d\xi\int_0^{2\pi}d\varphi\, v^4P_2(\xi)f_n(v,\xi,\varphi) \tag{26}$$

and $\lambda_i=v_T\tau_i$ is the Coulomb mean-free path, $\ln\Lambda$ is the Coulomb logarithm, and $\lambda_x=v_T/(n_iK_x)$ the CX mean-free path. The integral I_n will be evaluated in the next section. Note, that as long as the neutral density variation in the poloidal direction is weaker than the radial variation, we can assume that the self-similar solution f_n is approximately correct. Therefore we do not have to assume that the neutral density is constant in the poloidal direction, and we can keep I_n inside the flux-surface average in Eq. (25).

B. Modification of the parallel ion flow

The modification of the parallel ion flow due to neutrals is proportional to the integral I_n appearing in Eq. (25), which depends on the neutral distribution function.

If the neutral distribution is given by Eq. (7), with $\hat{f}(\mathbf{u})=\exp(-u^2)/\pi^{3/2}$, Eq. (26) becomes

$$I_n=\frac{4\beta n_n v_T^2}{\pi^{3/2}\gamma}\int_0^\infty u^3du\int_0^{\pi/2}dt\,P_2(\sin t)$$
$$\times\int_0^\pi\frac{d\varphi}{\sin\varphi}\int_0^1 ds$$
$$\times(e^{-u^2/s^2}e^{2(s-1)/(\gamma u\cos t\sin\varphi)}s^{-2\alpha}$$
$$+e^{-u^2s^2}e^{2(s-1)/(\gamma s u\cos t\sin\varphi)}s^{2\alpha-2}) \tag{27}$$

and by taking into account the identity $\int_0^\pi(d\varphi/\sin\varphi)e^{-a/\sin\varphi}=2K_0(a)$ where K_0 denotes the modified Bessel function of zeroth order, this integral can be rewritten as

$$I_n=\frac{8\beta n_n v_T^2}{\pi^{3/2}\gamma}\int_0^\infty u^3du\int_0^{\pi/2}dt\,P_2(\sin t)\int_0^1\frac{ds}{s^{2\alpha}}$$
$$\times\left(e^{-u^2/s^2}K_0\left(\frac{2(1-s)}{\gamma u\cos t}\right)\right.$$
$$\left.+e^{-u^2s^2}K_0\left(\frac{2(1-s)}{\gamma s u\cos t}\right)s^{4\alpha-2}\right). \tag{28}$$

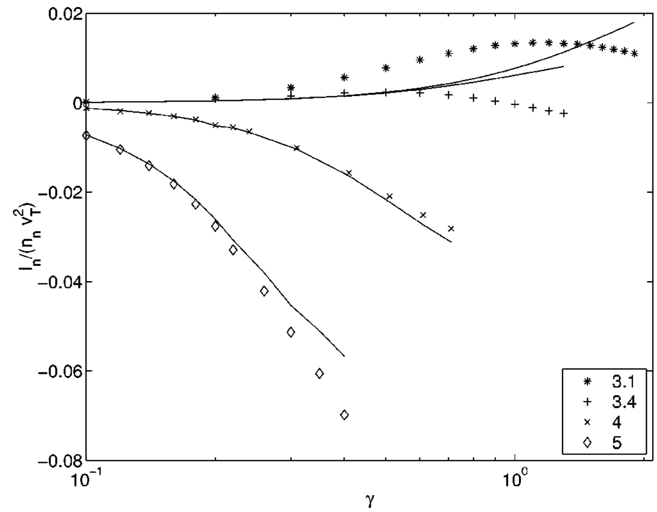

FIG. 6. $I_n/(n_nv_T^2)$ calculated from Eq. (28) as a function of γ, for different α (see inset) together with the short mean-free path approximation (continuous lines).

Numerical values of I_n calculated from Eq. (28) as a function of γ for different α's are shown in Fig. 6 together with the short mean-free path approximation for $0.5<\beta\leqslant 1$. Note that the short mean-free path approximation is reasonably good unless $\gamma\alpha$ is large.

C. Comparison with the short mean-free path regime

In the limit of short mean-free path, these results agree with those found earlier[5] by a Chapman–Enskog expansion. Indeed, the modification of the flow due to the distortion of the ion distribution function by the neutrals, V_n in Eq. (25), can be evaluated in the short mean-free path limit $(\gamma\rightarrow 0)$, in which the integral entering Eq. (25) is approximately

$$\lim_{\gamma\rightarrow 0} I_n=-\frac{(21-13\alpha+2\alpha^2)n_nv_T^2\beta\gamma^2}{8} \tag{29}$$

and V_n can be estimated as

$$V_n=\frac{5}{54}\frac{MB}{p_in_i}\frac{K_x\langle I_n\nabla_\parallel B\rangle}{\langle(\nabla_\parallel B)^2\rangle}$$
$$\sim-\frac{(21-13\alpha+2\alpha^2)}{43}\frac{n_n\beta\gamma^2qR}{n_i\lambda_x}v_T. \tag{30}$$

This agrees with the result we found in an earlier paper in the short mean-free path regime[5]

$$V_{\parallel i}=V_{\parallel i}^{\mathrm{neo}}+V_n^{smfp}, \tag{31}$$

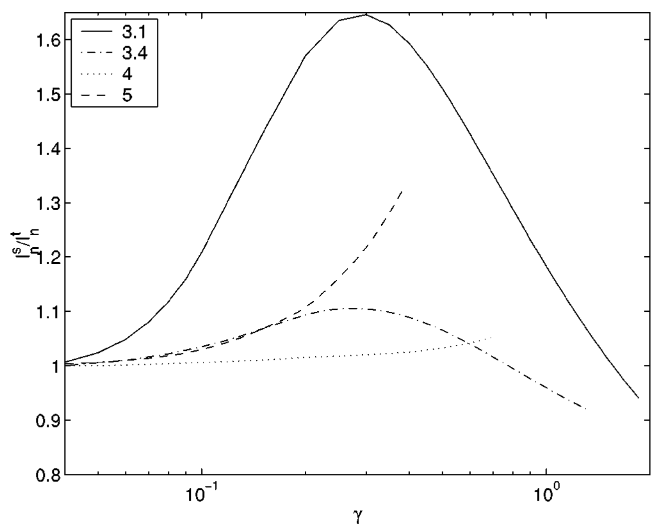

FIG. 7. I_n^t/I_n^s calculated from Eqs. (38) and (37) as a function of γ, for different α (see inset).

$$V_n^{smfp}=-\frac{5}{54}\frac{B\langle\nabla\cdot[K_x\beta(\nabla_\|B)\nabla(n_nT^2)]\rangle}{Mp_in_i\langle(\nabla_\|B)^2\rangle}. \tag{32}$$

Using $n_n=Nv_T^3/T^\alpha$ in Eq. (32) allows us to recover Eq. (30).

IV. EFFECT OF NEUTRAL ATOMS ON THE ELECTROSTATIC POTENTIAL AND ION FLOW

The radial electric field can be found from the steady state condition of no radial transport of toroidal momentum. Assuming there are no momentum sources,[6]

$$\langle R\hat{\varphi}\cdot(\boldsymbol{\pi}_i+\boldsymbol{\pi}_n)\cdot\nabla\psi\rangle=0, \tag{33}$$

where, because the neutral diffusivity is large,

$$\langle R\hat{\varphi}\cdot\boldsymbol{\pi}_i\cdot\nabla\psi\rangle\ll\langle R\hat{\varphi}\cdot\boldsymbol{\pi}_n\cdot\nabla\psi\rangle,$$

at the edge, just inside the separatrix if

$$1\lesssim\frac{n_nv_T\lambda_x}{n_iq^2\rho_i^2/\tau_i}\sim\frac{n_n}{n_i}10^4. \tag{34}$$

For $\langle R\hat{\varphi}\cdot\boldsymbol{\pi}_i\cdot\nabla\psi\rangle\sim\langle R\hat{\varphi}\cdot\boldsymbol{\pi}_n\cdot\nabla\psi\rangle$ the ion contribution must be retained as in Refs. 6 and 8.

In Eq. (33), the neutral viscosity is given by

$$\boldsymbol{\pi}_n=M\int d^3v(\mathbf{vv}-\mathbf{I}v^2/3)f_n \tag{35}$$

and can be evaluated using Eq. (7), where only the odd parts of the ion distribution will contribute to Eq. (33), so we may use Hazeltine's results[8] to make the replacement

$$\hat{f}(\mathbf{u})\to\frac{e^{-u^2}}{\pi^{3/2}}\left(\frac{MV_{\|i}}{T}-\frac{I}{15\Omega T}\frac{\partial T}{\partial\psi}\left(1-\frac{B^2}{\langle B^2\rangle}\right)\right.$$
$$\left.\times[u^2(1+2u^2)-20]\right)u\xi,$$

with $V_{\|i}$ given by Eq. (24). Note that we must assume that $V_{\|i}/T$ and $(\partial\ln T/\partial\psi)I/\Omega$ are constants over the edge region, in order to keep the right side of Eq. (3) in a self-similar form. Using the preceding, Eq. (33) takes the form

$$V_{\|i}=\frac{I\partial T/\partial\psi}{e\langle n_n\rangle}\left\langle\frac{n_n}{B}\left(1-\frac{B^2}{\langle B^2\rangle}\right)\right\rangle\frac{I_n^t}{I_n^s}, \tag{36}$$

where

$$I_n^s=\int_0^\infty u^4du\int_0^1d\xi\xi^2\int_0^\pi d\varphi\int_0^1\frac{ds}{s}$$
$$\times(e^{-u^2/s^2}e^{2(s-1)/(\gamma u\sqrt{1-\xi^2}\sin\varphi)}s^{-2\alpha}$$
$$-e^{-u^2s^2}e^{2(s-1)/(\gamma su\sqrt{1-\xi^2}\sin\varphi)}s^{2\alpha}) \tag{37}$$

and

$$I_n^t=\frac{1}{15}\int_0^\infty u^4du\int_0^1d\xi\xi^2\int_0^\pi d\varphi\int_0^1\frac{ds}{s^{2\alpha+1}}\left\{\left[\frac{u^2}{s^2}\right.\right.$$
$$\left.\times\left(1+2\frac{u^2}{s^2}\right)-20\right]e^{-u^2/s^2}e^{2(s-1)/(\gamma u\sqrt{1-\xi^2}\sin\varphi)}$$
$$-[u^2s^2(1+2u^2s^2)-20]$$
$$\left.\times e^{-u^2s^2}e^{2(s-1)/(\gamma su\sqrt{1-\xi^2}\sin\varphi)}s^{4\alpha}\right\}. \tag{38}$$

The radial electric field is affected by the neutrals since Eq. (36) gives

$$V_{\|i}^{\rm neo}+V_n=\frac{I\partial T/\partial\psi}{e\langle n_n\rangle}\left\langle\frac{n_n}{B}\left(1-\frac{B^2}{\langle B^2\rangle}\right)\right\rangle\frac{I_n^t}{I_n^s}. \tag{39}$$

Equation (39) may be rewritten as an equation determining the radial electric field given the ion density and temperature:

$$\frac{1}{p_i}\frac{\partial p_i}{\partial\psi}+\frac{Ze}{T}\frac{\partial\Phi}{\partial\psi}+\frac{1}{T}\frac{\partial T}{\partial\psi}\left[1.8\frac{B^2}{\langle B^2\rangle}+0.05\frac{B^2\langle(\nabla_\|\ln B)^2\rangle}{\langle(\nabla_\|B)^2\rangle}\right]$$
$$=\frac{M\Omega}{IT}\left(V_n-\frac{I\partial T/\partial\psi}{e\langle n_n\rangle}\left\langle\frac{n_n}{B}\left(1-\frac{B^2}{\langle B^2\rangle}\right)\right\rangle\frac{I_n^t}{I_n^s}\right). \tag{40}$$

The ratio of the two integrals in Eqs. (37) and (38) is of order unity. In the short mean-free path limit

$$\lim_{\gamma\to0}\frac{I_n^t}{I_n^s}=\frac{1+(17/5)(\alpha-3.5)(\alpha-4)\gamma^2}{1+(3/2)(\alpha-3.5)(\alpha-4)\gamma^2} \tag{41}$$

and as γ varies from much less than 1 to order unity, it increases if $\alpha<3.5$ or $\alpha>4$ and it decreases when $3.5<\alpha<4$. Figure 7 shows I_n^t/I_n^s versus γ for various α. Note that the value of I_n^t/I_n^s is rather close to 1 even for $\alpha=3.1$ and rather large mean-free paths. The I_n^t and I_n^s integrals converge as long as $\alpha>2.5$ and the ratio I_n^t/I_n^s is within a factor of 2 of the short mean-free path result unless $\alpha\to3$.

Equation (40) has the following noteworthy implication. In conventional neoclassical theory (without neutral atoms), the radial electric field is related to the radial gradients and parallel ion flow velocity by an equation of the form

$$\frac{Ze}{T}\frac{\partial\Phi}{\partial\psi}=\frac{M\Omega V_{\|i}}{IT}-\frac{1}{p_i}\frac{\partial p_i}{\partial\psi}-\frac{k}{T}\frac{\partial T}{\partial\psi}, \tag{42}$$

where the constant k depends on collisionality. In the Pfirsch–Schlüter regime

$$k=\left[1.8\frac{B^2}{\langle B^2\rangle}+0.05\frac{B^2\langle(\nabla_\parallel \ln B)^2\rangle}{\langle(\nabla_\parallel B)^2\rangle}\right].$$

If the neutral density is low, so that V_n is negligible, but still large enough that the viscosity of the neutrals is larger than that of the ions, then Eq. (40) shows that the radial electric field is given by Eq. (42) with $V_{\parallel i}$ given by Eq. (36). This replacement modifies the coefficient of the temperature gradient term by order unity. Notice that except for the temperature gradient term on the right-hand side of (36), the neutrals have effectively stopped the ions from rotating. In the conventional Pfirsch–Schlüter regime description, the neutral viscosity is neglected and Eq. (33) gives a different relation between the gradients of the potential and the temperature that does not explicitly involve the parallel ion flow.[7,8]

The importance of the neutral contribution to the ion flow can be estimated by

$$\frac{V_n}{V_{\parallel i}}\sim\frac{5}{27}\frac{a}{\lambda_x}\frac{W}{\rho_i}\frac{MI_n}{p_i}\left(\frac{R}{a}\right)^2\sim\frac{5}{54}\frac{a}{\rho_i}\left(\frac{R}{a}\right)^2\frac{n_n}{n_i}\beta\gamma\sim 10^3\frac{n_n}{n_i}. \tag{43}$$

Here we have used the following set of parameters: characteristic radial scale length $W=2$ cm, CX mean-free path $\lambda_x=1$ cm, Coulomb mean-free path $\lambda_i=1$ m, major radius $R=1$ m, aspect ratio $R/a=3$ and gyroradius $\rho_i=0.02$ cm.

Note that the modification of the parallel ion flow due to the distortion of the distribution function due to neutrals can be of importance when $n_n/n_i \gtrsim 10^{-3}$.

Equation (40) shows that the electrostatic potential is affected by the neutrals through the terms on the right-hand side. The first term on the right is proportional to the neoclassical flow modification by the neutrals, and depends on the neutral density. As we noted above, this term is significant when $n_n/n_i \gtrsim 10^{-3}$. However, the second term on the right is independent of the neutral density and will modify the electrostatic potential as long as the cross-field viscosity of the neutrals exceeds that of the ions. As noted in Eq. (34), this occurs even for very small neutral to ion density ratios. Notice, however, that the I_n^t/I_n^s coefficient of the second term is sensitive to the details of the neutral distribution function and becomes larger as $\alpha\to 3$ because of an extended neutral tail.

V. CONCLUSIONS

In conclusion, we have used a self-similar solution of the neutral kinetic equation to explore the effect of neutral atoms on edge plasma kinetics in the finite mean-free path regime. The choice of the similarity parameter α is limited by the physical requirement of finite neutral heat flux. Furthermore, the ratio of neutral mean-free path to the radial scale length cannot be allowed to be larger than $\gamma\simeq 2$, because that would lead to unrealistically small β values (meaning that the ionization dominates over CX).

We have shown that the modification of the ion flow due to CX with neutrals can be significant when $n_n/n_i \gtrsim 10^{-3}$. The ion flow calculated here agrees within a factor of 2 with the short mean-free path solution of Ref. 5 for $\gamma\alpha \lesssim 10$. Substantial departure of the self-similar solution from the short mean-free path limit arises in the neutral heat flux as $\alpha\to 3$ because this is a higher velocity moment of the neutral distribution function and therefore more sensitive to a tail in the neutral distribution function. The neutral viscosity modifications of the electrostatic potential are also typically of the same order of magnitude as found in the short mean-free path regime in Ref. 6, implying that when $n_n/n_i > 10^{-4}$ the neutrals determine the radial electric field just inside the separatrix if the tokamak edge is in the Pfirsch–Schlüter regime. Thus, it is more difficult for the neutrals to affect the ion flow than it is for them to determine the electrostatic potential at the edge.

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