

Controlling edge plasma rotation through poloidally localized refueling

P. Helander^{a)}

Euratom/UKAEA Fusion Association, Culham Science Centre, Abingdon, Oxfordshire OX14 3DB, United Kingdom

T. Fülöp

Department of Electromagnetics, Chalmers University of Technology, Göteborg, Sweden

Peter J. Catto

Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 6 June 2003; accepted 16 July 2003)

The transport of angular momentum due to neutral atoms in the tokamak edge is calculated and shown to be sensitive to the poloidal location of the neutrals. In the absence of external momentum sources, the edge plasma is predicted to rotate spontaneously in the opposite direction to the plasma current, at a speed proportional to the radial ion temperature gradient. If the plasma is collisional, this countercurrent rotation is largest if the neutrals are concentrated on the inboard side of the torus, while the opposite holds in a collisionless plasma. The presence of heavy impurity ions also promotes countercurrent rotation. The rotation caused by an external momentum source, such as neutral-beam injection, is found to be larger when the neutrals are on the inboard rather than the outboard side. © 2003 American Institute of Physics. [DOI: 10.1063/1.1616014]

I. INTRODUCTION

In the neoclassical theory of tokamak transport, the radial transport of toroidal angular momentum is very weak. According to most calculations,^{1–4} in the collisional regime this transport lacks the Pfirsch–Schlüter factor $2q^2$, with q the safety factor, while in the collisionless regime it lacks the banana enhancement factor, which is of order $\epsilon^{-3/2}$, with $\epsilon = r/R$ the inverse aspect ratio. Neoclassical momentum transport is thus typically at least an order of magnitude smaller than the corresponding energy transport. As a result, processes other than those included in standard neoclassical theory are likely to determine the toroidal rotation and radial electric field in an axisymmetric tokamak.

In the edge plasma, the presence of neutral atoms provides an efficient means of momentum transport. Although these particles are relatively few, their high cross-field mobility enables them to carry a substantial flux of momentum across the magnetic field. As shown in Refs. 5 and 6, if there is more than about one neutral atom in 10^4 ions, for typical edge plasma parameters, then the cross-field flux of momentum carried by neutrals is larger than the corresponding neoclassical flux. Since such neutral abundances are typical in the tokamak edge, one may expect that neutrals may influence toroidal rotation and the radial electric field at the plasma edge, at least if turbulent transport of momentum is relatively weak.

Experimentally, it has long been known that neutral atoms in the edge plasma affect access to the high-confinement H-mode in tokamaks. The first H-mode experiments required careful wall conditioning and low wall recycling,⁷ and a series of subsequent experiments on a range of tokamaks have

found neutrals to be important for the H-mode transition in various ways.^{8–15} In several of these studies,^{8,10,12–15} it was found that the poloidal location of recycling or injected neutrals can have a dramatic effect on H-mode access, which is often favored by fueling the plasma from the inboard side of the torus. For instance, in the Mega-Ampère Spherical Tokamak (MAST),¹⁵ Ohmic H-mode can *only* be achieved with inboard refueling, which also allows easier H-mode access in auxiliary heated discharges.

This background provides the motivation for the present paper, where we examine theoretically the effect of a poloidally localized neutral source on toroidal rotation and the radial electric field in the tokamak edge. The theoretical framework is that of standard neoclassical theory^{16,17} supplemented by a short-mean-free-path kinetic treatment of neutral transport of angular momentum.⁵ Among other things, the theory predicts that in steady state the edge plasma should rotate spontaneously in the toroidal direction at a speed determined by the ion temperature gradient and the poloidal distribution of the neutrals. Some of the results concerning collisional plasmas have recently been published in briefer form elsewhere.¹⁸ The intention of the present paper is to give a fuller picture by extending the analysis to allow for lower collisionality, the presence of impurity ions, and external momentum sources. These extensions should allow the theory to be compared with experiments over a wider range of plasma conditions. Such a comparison with data from MAST shows encouraging agreement,¹⁹ and a more detailed account will be published elsewhere.

The remainder of this paper is organized as follows. After some general background material presented in Sec. II, the case of a collisional (Pfirsch–Schlüter regime) plasma is considered in Sec. III, which partly duplicates Ref. 18 but

^{a)}Electronic mail: per.helander@ukaea.org.uk

extends the theory to hold for an impure plasma. Less collisional (banana regime) plasmas are treated in the following section, again allowing for the presence of impurities, and the effect of external momentum sources such as neutral beams is considered in Sec. V. The last section contains a discussion of these results from a physical point of view and their possible experimental implications.

II. GENERAL BACKGROUND

We consider the toroidal rotation of an axisymmetric plasma consisting of four species: electrons (e), bulk hydrogenic ions (i), their neutral counterparts (n), and heavy impurity ions (z) of charge $z \gg 1$. The impurity number density is assumed to be of order $n_z \sim n_e z^{-2}$, where n_e is the electron density, so that $Z_{\text{eff}} - 1 = n_z z^2 / n_e = O(1)$. The impurities are assumed to be in the collisional Pfirsch-Schlüter regime, while the bulk ions can be either collisional or collisionless. The magnetic field is given by

$$\mathbf{B} = I(\psi) \nabla \varphi + \nabla \varphi \times \nabla \psi,$$

where ψ denotes the poloidal flux and the toroidal angle φ runs in the direction of the plasma current, so that ψ increases towards the edge. Insofar as transport is much faster within flux surfaces than across them, as is usually assumed, the electron and ion densities and temperatures are flux functions (if the rotation is subsonic), and the plasma flow velocity must be of the form

$$\mathbf{V}_i = \omega(\psi) R \hat{\varphi} + u_{i\theta}(\psi) \mathbf{B}. \quad (1)$$

This form is the most general expression for a divergence-free velocity field tangential to flux surfaces. The flux functions $\omega(\psi)$ and $u_{i\theta}(\psi)$ are given by^{16,17,20}

$$\omega(\psi) = -\frac{d\Phi}{d\psi} - \frac{1}{n_i e} \frac{dp_i}{d\psi}, \quad (2)$$

$$u_{i\theta}(\psi) = -\frac{kI}{e \langle B^2 \rangle} \frac{dT_i}{d\psi}, \quad (3)$$

where $\Phi(\psi)$ is the electrostatic potential, $p_i = n_i T_i$ the bulk ion pressure, $\langle \dots \rangle$ denotes the flux-surface average, and the coefficient k depends on plasma collisionality. Note that the poloidal rotation is entirely represented by the term in Eq. (1) containing $u_{i\theta}$ and is independent of the radial electric field, while the toroidal rotation has contributions from both terms. The fundamental reason why a radial electric field does not affect poloidal rotation is that this field vanishes in a frame rotating toroidally at the angular frequency $-d\Phi/d\psi$. As long as this rotation is small enough not to give rise to large centrifugal or Coriolis forces, the equations governing the plasma are the same in the rotating frame and the laboratory frame. Since the poloidal rotation must be the same in the two frames, it cannot depend on the radial electric field.

The constant k is calculated in conventional neoclassical theory and is of order unity; explicit expressions are given below. As shown in Refs. 6 and 21, k is not much affected by the presence of neutrals unless their density n_n is so large that $n_n/n_i \gtrsim \rho_i/qR$, where ρ_i is the ion gyroradius. Such high neutral densities are uncommon in most tokamaks, ex-

cept very close to or beyond the separatrix. For simplicity, we shall assume that the neutral density is smaller, so that the ion distribution function is not much affected by the neutrals and can be taken from ordinary neoclassical theory. Other-wise modifications calculated in Refs. 6 and 21 must be retained.

The radial electric field appearing in Eq. (2) is determined by the transport equation for toroidal angular momentum, which is²⁰

$$\left\langle m_i R \frac{\partial(n_i V_{i\varphi} + n_n V_{n\varphi})}{\partial t} \right\rangle = \langle \mathbf{j} \cdot \nabla \psi \rangle - \langle R \hat{\varphi} \cdot \nabla \cdot (\boldsymbol{\pi}_i + \boldsymbol{\pi}_n) \rangle + \langle R F_\varphi \rangle, \quad (4)$$

when summed over all species. Here $\boldsymbol{\pi}_i$ and $\boldsymbol{\pi}_n$ are the ion and neutral viscosity tensors, \mathbf{j} is the current, and F_φ denotes any external force acting on the plasma, e.g., from neutral beams. In this expression, the viscosity term can be simplified somewhat by noting that, for any symmetric tensor $\boldsymbol{\pi}$ we have

$$R \hat{\varphi} \cdot (\nabla \cdot \boldsymbol{\pi}) = \nabla \cdot (R \hat{\varphi} \cdot \boldsymbol{\pi}) \quad (5)$$

since $\nabla(R \hat{\varphi}) = \hat{\mathbf{R}} \hat{\varphi} - \hat{\varphi} \hat{\mathbf{R}}$. In steady state there can be no radial current, and Eq. (4) becomes¹⁶

$$\frac{1}{V'} \frac{d}{d\psi} (V' \langle R \hat{\varphi} \cdot (\boldsymbol{\pi}_i + \boldsymbol{\pi}_n) \cdot \nabla \psi \rangle) = \langle R F_\varphi \rangle,$$

where $V(\psi)$ is the volume inside the flux surface labeled by ψ . Finally, if the neutral viscosity is larger than ion viscosity and if we restrict our attention to the edge where little beam momentum is absorbed, then we obtain

$$\langle R \hat{\varphi} \cdot \boldsymbol{\pi}_n \cdot \nabla \psi \rangle = \frac{1}{V'} \int \langle R F_\varphi \rangle dV, \quad (6)$$

where the volume integral is taken over the entire plasma and $dV = V' d\psi$. This is the equation we shall use to calculate the radial electric field in the next three sections.

The neglect of $\boldsymbol{\pi}_i$ in Eq. (6) is justified if the neutral viscosity is larger than both the neoclassical and anomalous ion viscosities. As already mentioned, the neoclassical ion viscosity is fairly small and can be neglected for realistic neutral densities at the edge, where typically⁶

$$\frac{n_n}{n_i} \gtrsim \frac{\epsilon^3 \lambda_i \rho_i^2}{(v_z \tau)^{1/2} q R^2 L_\perp}, \quad (7)$$

where ρ_i is the ion gyroradius, λ_i the Coulomb mean-free path, L_\perp the characteristic radial scale length of density and temperature variation, v_z is the ionization rate, and $\tau^{-1} = n_i \langle \sigma v \rangle_x \approx 2.93 n_i \sigma_x (T_i/m_i)^{1/2}$ the charge-exchange frequency.²² The neglect of anomalous viscosity may seem more questionable since the heat flux is certainly often turbulent. However, it should be remembered that the *particle* transport associated with neutrals in the edge is necessarily as large as that of all other mechanisms combined since every recycling ion that leaves the plasma because of collisional diffusion or turbulence makes its way back as a neu-

tral atom. Therefore, it is perhaps not implausible that something similar could hold for momentum transport. Moreover, experimental evidence mentioned in the Introduction suggests that neutrals do affect plasma rotation considerably. However, if the anomalous transport of angular momentum is larger than its neutral counterpart, then the radial electric field will be determined by turbulence rather than the processes considered in this paper.

The calculation of π_n requires solving the neutral kinetic equation.²³ To do so analytically, we employ two approximations: the neutral mean-free path with respect to charge exchange is taken to be independent of velocity and shorter than L_\perp . The short-mean-free-path approximation is not very accurate in many situations, especially in the H-mode, but should give qualitatively correct results. Moreover, in Ref. 18 the neutral viscosity calculated in this way was compared with that from a full solution of the neutral kinetic equation obtained for a special class of self-similar plasma profiles and was then found to be surprisingly accurate, even when the mean-free path was comparable to the plasma scale length. In the short-mean-free-path approximation, the neutrals undergo a random-walk with small steps through the edge plasma, and the neutral viscosity tensor becomes⁵

$$\pi_n = -\tau \nabla \cdot \left(\frac{m_i n_n}{n_i} \int \mathbf{v} \mathbf{v} f_i d^3 v \right) + O(\pi_i n_n / n_i) + \text{isotropic terms.} \tag{8}$$

Taking the ion distribution function f_i in various collisional regimes from neoclassical theory, this result allows us to calculate the neutral viscosity and hence the radial electric field from Eqs. (1), (2), and (6).

III. PFIRSCH-SCHLÜTER REGIME

The plasma just inside the separatrix is often in the collisional Pfirsch-Schlüter regime, where it is appropriate to expand the ion distribution function in Sonine polynomials as

$$f_i = f_{i0} + \frac{m_i \mathbf{v}}{T_i} \cdot \left[\mathbf{V}_i + \left(x^2 - \frac{5}{2} \right) \frac{2 \mathbf{q}_i}{5 p_i} + L_2^{(3/2)}(x^2) \frac{8 q_{i\parallel} v_\parallel \mathbf{B}}{75 p_i B} + \dots \right] f_{i0}. \tag{9}$$

Here $f_{i0} = (m_i / 2 \pi T_i)^{3/2} \exp(-x^2)$ is the Maxwellian, $x^2 = m_i v^2 / 2 T_i$, $L_2^{(3/2)}(x^2) = (x^4 - 7x^2 + 35/4) / 2$, and \mathbf{q}_i is the ion heat flux given below. The last term in Eq. (9) does not contribute to the viscosity (8), of which the desired component (6) becomes^{5,18}

$$\langle R \hat{\boldsymbol{\phi}} \cdot \pi_n \cdot \nabla \psi \rangle = -\tau \left\langle \nabla \psi \cdot \nabla \left[R T_i n_n \left(V_{i\phi} + \frac{2 q_{i\phi}}{5 p_i} \right) \right] \right\rangle. \tag{10}$$

Since this expression represents the radial transport of angular momentum, it is not surprising that the first term on the right-hand side contains the product of the neutral angular momentum $m_i n_n R V_{i\phi}$ and the neutral diffusion coefficient

$D \sim \tau T_i / m_i$. The second term, which is related to the toroidal heat flux, is given a physical interpretation in Sec. VI. In the derivation of this result it is assumed that plasma parameters such as density and temperature vary more rapidly in the radial direction than does the magnetic field.

Our problem is thus reduced to calculating the toroidal ion particle and heat fluxes. To do so for an impure plasma is most conveniently (although somewhat approximately) done using the Hirshman-Sigmar moment formalism,¹⁷ and is particularly simple in the present case of a plasma with small impurity concentration, $n_z \ll n_i$. The ion heat flux is then of the same form as in a pure hydrogen plasma,²⁴

$$\mathbf{q}_i = -\frac{5 p_i}{2 e} \frac{d T_i}{d \psi} R \hat{\boldsymbol{\phi}} + q_{i\theta} \mathbf{B}, \tag{11}$$

$$q_{i\theta} = \frac{5 I p_i}{2 e \langle B^2 \rangle} \frac{d T_i}{d \psi}.$$

The reason for this is that if the impurities are few, $n_z \ll n_i$, they do not carry much heat flux, so the total ion heat flux therefore remains the same as in a pure plasma. The poloidal rotation is obtained from the constraint $\langle \mathbf{B} \cdot \nabla \cdot (\boldsymbol{\pi}_i + \boldsymbol{\pi}_z) \rangle = 0$,¹⁷ but again the impurities are too few to make a significant contribution to the parallel viscosity if $n_z \ll n_i$, so we have^{17,20}

$$\langle \mathbf{B} \cdot (\nabla \cdot \boldsymbol{\pi}_i) \rangle = \langle (\nabla_\parallel B)^2 \rangle \left(\mu_{i1} u_{i\theta} + \mu_{i2} \frac{2 q_{i\theta}}{5 p_i} \right) = 0.$$

However, since $n_z z^2 / n_i = O(1)$, the impurities do affect the neoclassical parallel viscosity coefficients μ_{i1} and μ_{i2} , which contain information about the collision operator describing both ion-ion and ion-impurity collisions. This implies that $u_{i\theta}$ is of the form (3), with $k = \mu_{i2} / \mu_{i1}$. In the limit of no impurities, $Z_{\text{eff}} = 1$, their ratio is $\mu_{i2} / \mu_{i1} \approx 1.7$, while a more accurate calculation of poloidal rotation by Hazeltine²⁵ gave $k = 1.8 + 0.05 \langle B^2 \rangle \langle (\nabla_\parallel \ln B)^2 \rangle / \langle \nabla_\parallel B \rangle^2$. In the opposite limit of high impurity content, $Z_{\text{eff}} \gg 1$, ion-ion collisions may be ignored and it is a simple matter to calculate $\mu_{i2} / \mu_{i1} = 5/2$.²⁶ For intermediate impurity concentration, interpolation formulas are available in the literature for the neoclassical viscosity coefficients but none of these appear to reproduce both these limits correctly. A simple formula that does this (neglecting the small term proportional to 0.05) is, e.g.,

$$k = \frac{5}{2} - \frac{0.7}{Z_{\text{eff}}},$$

and we shall use this expression in our numerical results below.

For an Ohmic plasma with localized gas puffing where most neutrals are in one place poloidally, say at $\theta = \theta_*$, Eqs. (6) and (10) imply

$$\frac{d}{d \psi} \left[R T_i n_n \left(V_{i\phi} + \frac{2 q_{i\phi}}{5 p_i} \right) \right]_{\theta = \theta_*} = 0$$

since there is no external momentum input, $F_\varphi=0$. Assuming that the rotation stays finite in the core where $n_n \rightarrow 0$ gives $V_\varphi = -2q_i\varphi/5p_i$ at $\theta = \theta_*$, and it follows from Eqs. (1), (3), and (11) that

$$\omega(\psi) = \frac{1}{e} \frac{dT_i}{d\psi} \left(1 + \frac{(k-1)I^2}{\langle B^2 \rangle R_*^2} \right), \quad (12)$$

where $R_* = R(\psi, \theta_*)$ is the major radius at the puffing location. The toroidal rotation at an arbitrary position, of major radius R , on the flux surface in question is then given by

$$V_{i\varphi} = \frac{I^2}{e \langle B^2 \rangle R} \frac{dT_i}{d\psi} F_V(R, R_*), \quad (13)$$

with

$$F_V(R, R_*) = k \left(\frac{R^2}{R_*^2} - 1 \right) + \frac{\langle B^2 \rangle R^2}{I^2} - \frac{R^2}{R_*^2}. \quad (14)$$

Thus, the absence of external momentum sources does not imply that the plasma should not rotate. Because there is a drive term in Eq. (10) involving the toroidal heat flux $q_{i\varphi}$, the plasma starts rotating at a speed proportional to the radial ion temperature gradient. Since $dT_i/d\psi$ is normally negative the rotation is in the direction opposite to that of the plasma current, and it is subsonic if the temperature gradient scale length exceeds the poloidal ion gyroradius. The rotation is caused by the presence of neutrals, but is independent of the neutral density as long as the neutral cross-field viscosity dominates over its neoclassical and turbulent counterparts. A physical picture of how the rotation arises is discussed in Sec. VI.

The rotation velocity (13) depends strongly on the puffing location R_* . In a tokamak with circular (or elliptical) cross section and small inverse aspect ratio, $\epsilon \ll 1$, so that $R = R_0(1 + \epsilon \cos \theta)$ and $B = B_0(1 - \epsilon \cos \theta)$, Eq. (13) becomes

$$V_{i\varphi} \approx \frac{2\epsilon}{eB_\theta} \frac{dT_i}{dr} [k \cos \theta - (k-1) \cos \theta_*].$$

The rotation speed in the outer midplane, say, is thus larger with inboard puffing ($\theta_* = \pi$) than with outboard puffing ($\theta_* = 0$) by a factor $2k-1$, which is in the range between 2.6 and 4. Note that this relative difference between inboard and outboard puffing is independent of ϵ while the absolute value of the rotation becomes small in the limit $\epsilon \rightarrow 0$. These conclusions remain qualitatively valid if Eq. (13) is evaluated for magnetic equilibria corresponding to actual tokamaks. Figure 1 shows the normalized rotation speed $F_V(R, R_*)$ with R corresponding to the outer midplane as a function of the poloidal location of the neutrals, θ_* , for typical edge equilibria in the spherical tokamak MAST and the more conventional tokamak Alcator C-Mod. As can be seen from this figure, the rotation is larger at small aspect ratio, and there is a significant difference between inboard and outboard puffing in both machines. The presence of impurities enhances this difference.

All of this reflects that fact that the radial electric field depends on the poloidal location of the neutrals. The electric field is obtained from Eqs. (2) and (12), which give

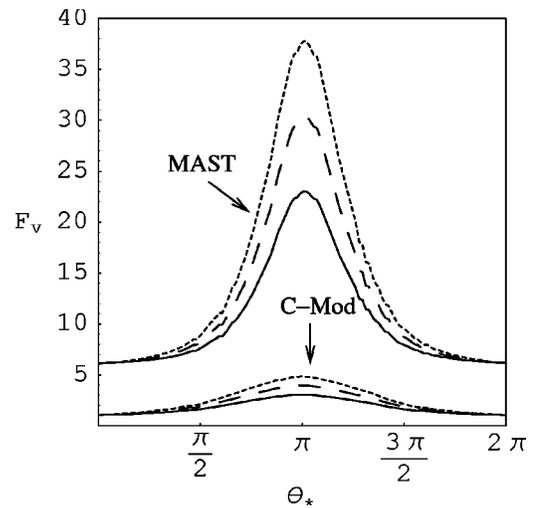


FIG. 1. Normalized toroidal outboard rotation as a function of poloidal neutral location in the Pfirsch-Schlüter regime. $\theta_* = 0$ corresponds to the outboard midplane, $\theta_* = \pi$ to the inboard midplane. The upper three curves are for a typical magnetic equilibrium in MAST and the lower ones are for Alcator C-Mod. The solid curves correspond to $Z_{\text{eff}}=1$, the dashed ones to $Z_{\text{eff}}=2$, and the dotted ones to $Z_{\text{eff}} \geq 1$.

$$-\frac{d\Phi}{d\psi} = \frac{T_i}{n_i e} \frac{dn_i}{d\psi} F_E(R_*),$$

with

$$F_E(R_*) = 1 + \eta_i \left(2 + \frac{(k-1)I^2}{\langle B^2 \rangle R_*^2} \right),$$

where $\eta_i = d \ln T_i / d \ln n_i$ is the ratio between the temperature and density gradient. Note that the edge electric field is predicted to be inward, as is usually observed in experiments. Figures 2 and 3 show F_E in the same magnetic configurations as Fig. 1, for various values of η_i and Z_{eff} . The radial electric field is largest when: (i) the neutrals are located on the inboard side; (ii) when η_i is large; (iii) when the aspect

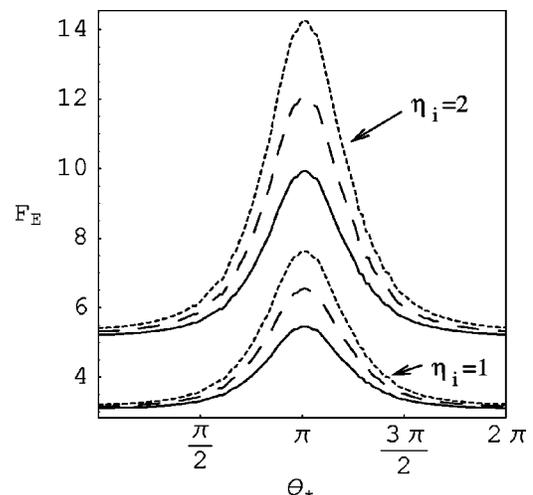


FIG. 2. Normalized radial electric field as a function of poloidal neutral location in MAST in the Pfirsch-Schlüter regime, for $Z_{\text{eff}}=1$ (solid), $Z_{\text{eff}}=2$ (dashed), and $Z_{\text{eff}} \geq 1$ (dotted). The lower curves correspond to $\eta_i=1$, and upper curves to $\eta_i=2$.

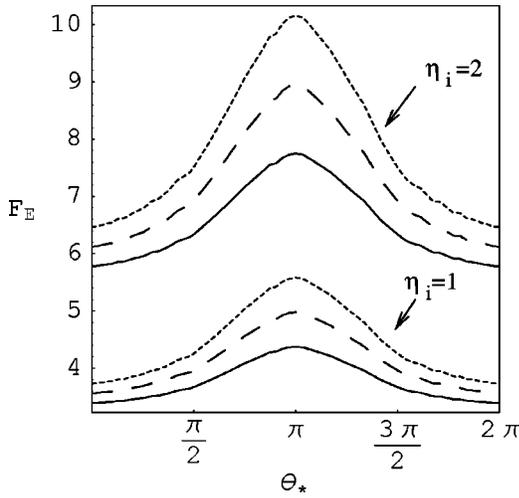


FIG. 3. Same as Fig. 2 but for Alcator C-Mod.

ratio is low; and (iv) when the impurity content is high. The results shown in Figs. 1–3 are fairly insensitive to the particular choice of magnetic equilibrium and vary very little between different discharges.

IV. BANANA REGIME

Even if the plasma closest to the separatrix is collisional in many tokamaks, it often becomes collisionless some short distance into the plasma where the temperature is higher. The plasma may also be collisionless all the way up to the edge. Either case calls for an evaluation of the neutral viscosity in the banana regime of low collisionality. In this regime, the ion distribution function can only be calculated exactly in the limits of very large or very small aspect ratio, or in the limit $Z_{\text{eff}} \rightarrow \infty$. In intermediate regions, approximate expressions must be used. The most accurate such analytical expression in the literature is^{20,27}

$$f_i = (1 - \boldsymbol{\rho} \cdot \nabla) f_{i0} + F + g, \tag{15}$$

where the term containing the gyroradius vector $\boldsymbol{\rho} = \mathbf{B} \times \mathbf{v} / \Omega_i B$ is the diamagnetic correction to the Maxwellian f_{i0} , and the other two terms are

$$F = - \frac{I v_{\parallel}}{\Omega_i} \frac{\partial f_{i0}}{\partial \psi} = - \frac{I v_{\parallel}}{\Omega_i} \left[\frac{d \ln p_i}{d \psi} + \frac{e}{T_i} \frac{d \Phi}{d \psi} + \left(x^2 - \frac{5}{2} \right) \frac{dT_i}{d \psi} \right] f_{i0}$$

and

$$g = \frac{m_i H(\lambda_c - \lambda) U B_0}{f_c T_i} \left[u_{i\theta} + \left(x^2 - \frac{5}{2} \right) \frac{2q_{i\theta}}{5p_i} \right] f_{i0}.$$

Here

$$f_c = 1 - f_i = \frac{3B_0^2}{4} \int_0^{B_{\text{max}}^{-1}} \frac{\lambda d\lambda}{\langle \sqrt{1 - \lambda B} \rangle}$$

is the ‘‘effective fraction’’ of circulating particles,¹⁷ $\Omega_i = eB/m_i$, $B_0^2 = \langle B^2 \rangle$, $\lambda = v_{\perp}^2/v^2 B$, $\lambda_c = B_{\text{max}}^{-1}$, with B_{max} the maximum value of B on the magnetic surface ψ ,

$$U = \frac{\sigma v B_0}{2} \int_{\lambda}^{\lambda_c} \frac{d\lambda'}{\langle \sqrt{1 - \lambda' B} \rangle},$$

and $\sigma = v_{\parallel}/|v_{\parallel}|$. The coefficients $u_{i\theta}$ and $q_{i\theta}$ are again to be calculated using the Hirshman–Sigmar moment formalism. This calculation can be done relatively easily in the case of a pure plasma, as in Refs. 20 and 27. If the plasma contains impurities, the calculation is more difficult but has recently been carried out with the result²⁸

$$\begin{pmatrix} u_{i\theta} \\ 2q_{i\theta} \\ 5p_i \end{pmatrix} = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} \frac{I}{eB_0^2} \frac{dT_i}{d\psi},$$

$$a_0 = \frac{1.17 + 3.64\alpha + 1.99\alpha^2}{1 + 0.46y + (2.58 + 1.65y)\alpha + 1.33(1 + y)\alpha^2},$$

$$a_1 = \frac{1 + 1.88\alpha}{1.17 + 2.82\alpha} a_0,$$

where $y = f_t/f_c$ and $\alpha = Z_{\text{eff}} - 1$. As in the case of the present paper, this calculation assumes the presence of a single, highly charged, collisional impurity in an otherwise pure hydrogen plasma.

Knowledge of the distribution function (15) now allows us to calculate the appropriate component (6) of the neutral viscosity (8). The diamagnetic part of the distribution function (15), which is

$$f_i^{(\text{dia})} = -\boldsymbol{\rho} \cdot \nabla f_{i0} = \frac{m_i \mathbf{v}}{T_i} \cdot \left[\mathbf{V}_{\wedge} + \left(x^2 - \frac{5}{2} \right) \frac{2\mathbf{q}_{\wedge}}{5p_i} \right] f_{i0},$$

where the diamagnetic particle and heat fluxes are

$$\mathbf{V}_{\wedge} = \frac{T_i}{eB^2} \left(\frac{d \ln p_i}{d \psi} + \frac{e}{T_i} \frac{d \Phi_i}{d \psi} \right) (\mathbf{I}B - RB^2 \hat{\boldsymbol{\phi}}),$$

$$\frac{2\mathbf{q}_{\wedge}}{5p_i} = \frac{1}{eB^2} \frac{dT_i}{d\psi} (\mathbf{I}B - RB^2 \hat{\boldsymbol{\phi}}),$$

is of the same form as the Pfirsch–Schlüter distribution (9), and the corresponding neutral viscosity is therefore similar to Eq. (10),

$$\begin{aligned} \langle R \hat{\boldsymbol{\phi}} \cdot \boldsymbol{\pi}_n \cdot \nabla \psi \rangle^{(\text{dia})} &= -\tau \left\langle \nabla \psi \cdot \nabla \left[RT_i n \left(\mathbf{V}_{\wedge \varphi} + \frac{2q_{\wedge \varphi}}{5p_i} \right) \right] \right\rangle \\ &\approx \tau \frac{d}{d\psi} \left\langle \frac{n_n T_i^2 |\nabla \psi|^4}{eB^2} \left(\frac{d \ln p_i}{d \psi} + \frac{e}{T_i} \frac{d \Phi_i}{d \psi} + \frac{dT_i}{d \psi} \right) \right\rangle. \end{aligned} \tag{16}$$

The neoclassical terms in the distribution function give rise to a contribution to the neutral viscosity equal to

$$\begin{aligned}
 \langle R \hat{\boldsymbol{\varphi}} \cdot \boldsymbol{\pi}_n \cdot \nabla \psi \rangle^{(\text{neo})} &\approx -\tau \frac{d}{d\psi} \left\langle \frac{n_n}{n_i} \int m_i (R \hat{\boldsymbol{\varphi}} \cdot \mathbf{v}) (\mathbf{v} \cdot \nabla \psi)^2 \right. \\
 &\quad \left. \times (F + g) d^3v \right\rangle \\
 &= -\tau \frac{d}{d\psi} \left\langle \frac{m_i n_n |\nabla \psi|^2 I}{2 n_i B} \int v_{\parallel} v_{\perp}^2 \right. \\
 &\quad \left. \times (F + g) d^3v \right\rangle, \tag{17}
 \end{aligned}$$

where we have again used Eq. (5) and assumed that the density and temperature vary more rapidly than the magnetic field. The first term in the integral is

$$\int F v_{\parallel} v_{\perp}^2 d^3v = -\frac{2 I n_i T_i^2}{m_i^2 \Omega_i} \left(\frac{d \ln p_i}{d\psi} + \frac{e}{T_i} \frac{d\Phi}{d\psi} + \frac{d \ln T_i}{d\psi} \right),$$

and the second one is

$$2\pi B^2 \int_0^{\infty} v^5 dv \int_0^{\lambda_c} g \lambda d\lambda = \frac{2 n_i T_i B^2 f_2}{m_i B_0 f_c} \left(u_{i\theta} + \frac{2 q_{i\theta}}{5 p_i} \right),$$

with

$$f_2 = \frac{15 B_0^3}{16} \int_0^{\lambda_c} \frac{\lambda^2 d\lambda}{\langle \sqrt{1 - \lambda B} \rangle},$$

so that

$$\begin{aligned}
 \int (F + g) v_{\parallel} v_{\perp}^2 d^3v &= -\frac{2 I n_i T_i^2}{m_i \Omega_i} \left[\frac{d \ln p_i}{d\psi} + \frac{e}{T_i} \frac{d\Phi}{d\psi} \right. \\
 &\quad \left. + \frac{d \ln T_i}{d\psi} - (a_0 + a_1) \right. \\
 &\quad \left. \times \frac{f_2}{f_c} \left(\frac{B}{B_0} \right)^3 \frac{d \ln T_i}{d\psi} \right].
 \end{aligned}$$

Adding the diamagnetic (16) and the neoclassical (17) contributions to the neutral viscosity thus gives

$$\begin{aligned}
 \langle R \hat{\boldsymbol{\varphi}} \cdot \boldsymbol{\pi}_n \cdot \nabla \psi \rangle &\approx \tau \frac{d}{d\psi} \left\langle \frac{n_n T_i^2 I^2 R^2 B_p^2}{e B^2} \left[\frac{B^2 R^2}{I^2} \left(\frac{d \ln p_i}{d\psi} \right. \right. \right. \\
 &\quad \left. \left. + \frac{e}{T_i} \frac{d\Phi}{d\psi} + \frac{d \ln T_i}{d\psi} \right) \right. \\
 &\quad \left. \left. - (a_0 + a_1) \frac{f_2}{f_c} \left(\frac{B}{B_0} \right)^3 \frac{d \ln T_i}{d\psi} \right] \right\rangle, \tag{18}
 \end{aligned}$$

where $B_p = |\nabla \psi|/R$ is the poloidal field strength.

Hence it follows that for a plasma without external momentum sources and with most neutrals concentrated at $R = R_*$, Eq. (6) implies

$$\frac{d \ln p_i}{d\psi} + \frac{e}{T_i} \frac{d\Phi}{d\psi} = \left[\frac{f_2}{f_c} (a_0 + a_1) \frac{B_* I^2}{B_0^3 R_*^2} - 1 \right] \frac{d \ln T_i}{d\psi},$$

with $B_* = B(R_*)$ the magnetic field strength at the puffing location. On the other hand, the toroidal flow velocity is

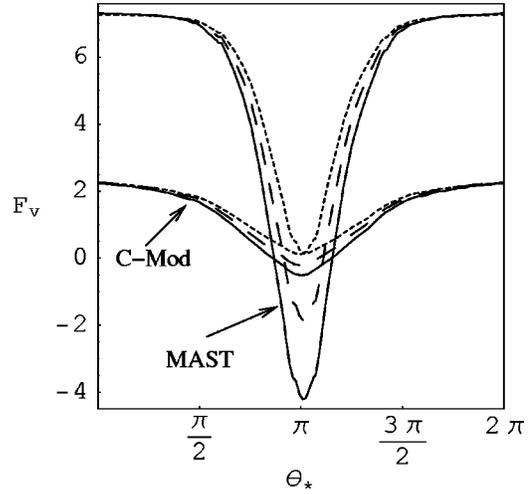


FIG. 4. Same as Fig. 1 but for the banana regime.

$$V_{i\varphi} = -\frac{R T_i}{e} \left(\frac{d \ln p_i}{d\psi} + \frac{e}{T_i} \frac{d\Phi}{d\psi} \right) + \frac{a_0 I^2}{e B_0^2 R} \frac{d T_i}{d\psi}.$$

The rotation velocity can thus be written in the same form as (13),

$$V_{i\varphi} = \frac{I^2}{e B_0^2 R} \frac{d T_i}{d\psi} F_V(R, R_*),$$

with F_V equal to

$$F_V(R, R_*) = \left(\frac{B_0 R}{I} \right)^2 + a_0 - (a_0 + a_1) \frac{f_2}{f_c} \frac{R^2 B_*}{R_*^2 B_0} \tag{19}$$

in the banana regime.

As in the Pfirsch–Schlüter regime, this result implies that the rotation velocity depends on the puffing location. Figure 4 shows how the normalized rotation F_V in the outboard midplane varies with puffing location in MAST and Alcator C-Mod. With outboard puffing the rotation is again in the countercurrent direction, but the rotation speed now becomes smaller if the neutral source is moved towards the inboard side. Although unlikely to be the case in practice, it even reverses if all the neutrals are located very close to the inboard midplane and the impurity content is low. Figures 5 and 6 illustrate the corresponding behavior of the electric field, which always points inwards and is largest with outboard puffing and is given by

$$-\frac{d\Phi}{d\psi} = \frac{T_i}{n_i e} \frac{dn_i}{d\psi} F_E(R_*),$$

$$F_E(R_*) = 1 + \eta_i \left(2 - (a_0 + a_1) \frac{f_2}{f_c} \frac{B_* I^2}{B_0^3 R_*^2} \right).$$

V. EFFECT OF AN EXTERNAL MOMENTUM SOURCE

As we have seen, the presence of neutral atoms makes the edge plasma rotate toroidally, even if there is no active external momentum input such as neutral-beam injection. If such a momentum source is present, it will modify the rotation and this modification will depend on the poloidal loca-

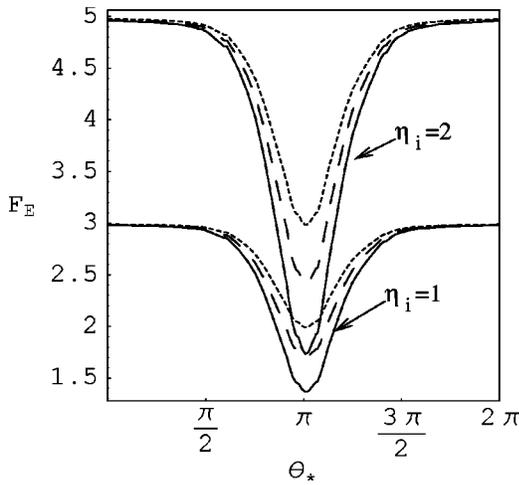


FIG. 5. Same as Fig. 2 but for the banana regime.

tion of the neutrals since their viscosity has such a dependence. It is important to note that, according to Eq. (6), the rotation of the edge plasma is affected by a momentum source even if this source happens to vanish locally in the edge. Of course, neutral beams are usually mostly aborted in the core of the plasma. In this case the integral on the right-hand side of Eq. (6) is constant and represents the total angular momentum deposited in the plasma, all of which must flow across the flux surfaces at the edge in steady state.

In the Pfirsch-Schlüter regime, where the viscosity is given by Eq. (10), the rotation velocity is determined by

$$\tau \left\langle \nabla \psi \cdot \nabla \left[RT_i n_n \left(V_{i\varphi} + \frac{2q_{i\varphi}}{5p_i} \right) \right] \right\rangle = - \frac{1}{V'} \int \langle RF_\varphi \rangle dV \equiv - \frac{S}{V'},$$

where S denotes the total angular momentum deposited in the plasma per unit time. We again take most neutrals to be in one place poloidally, $\theta = \theta_*$, and we write the toroidal rotation in this place as

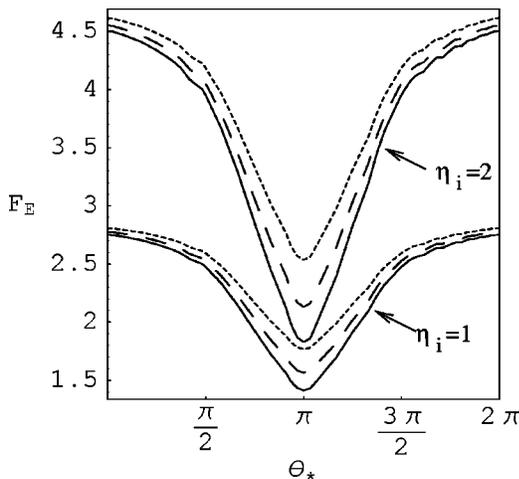


FIG. 6. Same as Fig. 3 but for the banana regime.

$$V_{i\varphi} = - \frac{2q_{i\varphi}}{5p_i} + U_*.$$

Then we obtain

$$R_*^3 B_{p*}^2 \frac{d(n_n T_i U_*)}{d\psi} = - \frac{S}{\tau V'},$$

which can be integrated to yield

$$U_*(\psi) = \frac{S}{n_n(\psi) T_i(\psi) R_*^3 B_{p*}^2} \int_{\psi}^{\psi_0} \frac{d\psi'}{\tau(\psi') V'(\psi')} + U_0,$$

where ψ_0 is the ψ at the last closed flux surface, and U_0 is an integration constant. Comparing this result with Eqs. (1)–(3) gives

$$\omega(\psi) = \frac{1}{e} \frac{dT_i}{d\psi} \left(1 + \frac{(k-1)I^2}{B_0^2 R_*^2} \right) + \frac{U_*}{R_*},$$

and the toroidal rotation velocity thus finally becomes

$$V_{i\varphi} = \frac{I^2}{e B_0^2 R} \frac{dT_i}{d\psi} F_V(R, R_*) + \frac{SR}{n_n(\psi) T_i(\psi) R_*^4 B_{p*}^2} \int_{\psi}^{\psi_0} \frac{d\psi'}{\tau(\psi') V'(\psi')} + U_0, \tag{20}$$

where the first term is similar to that found without momentum sources, Eq. (13). As expected, the additional term related to S (which tends to dominate for typical injection powers) causes the plasma to rotate in the direction of the source. It is interesting to note that this term, too, depends sensitively on the poloidal location of the neutrals, primarily due to the factor R_*^4 in the denominator. An entirely analogous expression is obtained in the banana regime, by combining Eqs. (6) and (18), again giving the rotation speed as in Eq. (20), but now with F_V defined by Eq. (19) rather than by Eq. (14).

A couple of comments are in order. First, we only expect these expressions to hold in the edge where there are enough neutrals present to satisfy Eq. (7). In the core, where $n_n \rightarrow 0$, some other mechanism must be responsible for angular momentum transport. Second, the integration constant U_0 is in principle determined by the boundary condition at the last closed flux surface, but this may be difficult in practice. In MAST, for example, the theory is only expected to be valid a few cm into the plasma since the neutral density is too high at the separatrix. As already remarked, a high enough neutral concentration affects the ion distribution function, which we have taken to be entirely neoclassical. It may also be the case that other processes not accounted for in ordinary neoclassical theory could be important very close to the edge, such as orbit losses and other effects associated with finite ion orbits. It may therefore be difficult to determine U_0 and obtain an absolute prediction for the rotation velocity, but Eq. (20) nevertheless suggests that this rotation should be substantially different with inboard and outboard refueling.

The inverse dependence on $R_*^4 B_{p*}^2$ in Eq. (20) can be understood by considering flux surfaces rotating as rigid bodies, with a rotation speed $V_\varphi = \omega(\psi)R$. A factor $R_*^2 B_{p*}^2$

arises because the spacing between adjacent flux surfaces is inversely proportional to $|\nabla\psi|=RB_p$ and the neutral diffusion coefficient is proportional to the square of the mean-free path. The remaining factor R_*^2 reflects the circumstance that the angular momentum carried by each neutral is equal to $m_i\omega R_*^2$.

VI. DISCUSSION

As we have seen in the previous three sections, if neutral atoms are responsible for a major part of the angular momentum transport in the tokamak edge, then the toroidal rotation and radial electric field should be sensitive to the poloidal location of these neutrals. This sensitivity is not surprising in the case when momentum is injected into the plasma by neutral beams or RF waves. The ability of neutral atoms to carry angular momentum out of the plasma clearly increases with the major radius, so the rotation should be highest if the neutrals are concentrated on the inboard side of the torus.

Perhaps more surprising is the conclusion that even if there is no apparent input of external momentum into the plasma, the neutrals nevertheless cause it to rotate toroidally. This rotation is proportional to the ion temperature gradient and is in the opposite direction to the plasma current in the Pfirsch–Schlüter regime. The reason for this rotation is that the neutral viscosity (10) is not just related to the rotation speed $V_{i\varphi}$ but also to the toroidal heat flux $q_{i\varphi}$, which thus acts as a drive for rotation. On its own, the term containing $V_{i\varphi}$ would damp any rotation, just like ordinary viscosity in a simple fluid. However, the term containing $q_{i\varphi}$, which is proportional to $dT_i/d\psi$, drives toroidal rotation. Physically, this may be understood by considering the flux of momentum between two neighboring flux surfaces, A and B, say. Ordinary viscosity operates if there is a parallel (or toroidal) particle flux on A but not on B. In addition to flowing along the field, the particles on A also perform a random walk in the radial direction. Some of the parallel momentum on A therefore spills over to B, which implies that there is radial transport of parallel momentum. To understand the term in the viscosity that is proportional to the toroidal heat flux $q_{i\varphi}$, suppose that there are two kinds of particles on A: fast ones and slow ones. The fast ones are assumed to move in one direction (the “positive” direction) and the slower ones in the opposite direction, but let us suppose that there are more slow particles than fast ones so that there is no net parallel particle flux on A. There is then, however, a heat flux in the positive direction on A. Now, if the fast particles perform a radial random walk that is faster than that of the slow ones, there will be a net flux of positive momentum from A to B. Flux surface B will start rotating although A does not. A parallel *particle* flux arises on B as a consequence of the *heat* flux on A, and since this heat flux is related to the radial temperature gradient by neoclassical theory, such a gradient drives plasma rotation. A similar heat-flux-driven contribution to the viscosity arises in a fully ionized plasma flowing at a speed comparable to the diamagnetic velocity^{29,30} and can be interpreted in similar terms.

Thus, the mechanism causing the edge plasma to rotate—in spite of axisymmetry and momentum

conservation—can be summarized as follows. Although the plasma is axisymmetric, the magnetic field breaks the toroidal symmetry by introducing a heat flux (11) with a preferred direction. As we have just discussed, this enables neutral atoms to transfer angular momentum between flux surfaces. If the plasma did not interact with the outside world, this would cause some flux surfaces to rotate one way and others to rotate the other way, but on a radial average there would be no overall rotation because of angular momentum conservation. However, since the partially ionized plasma *does* exchange momentum with the vessel wall, by ion losses and recycling, net momentum can be imparted to the plasma, which “pushes off” the wall and starts rotating in the opposite direction to the toroidal heat flux.

As mentioned in the Introduction, the poloidal location of edge neutrals has been observed to affect H-mode access in many tokamaks. It is tempting to speculate that the results derived here may be related to these observations. The turbulence thought to be responsible for the poor confinement in L-mode usually “balloons” on the outboard side of the torus, and it is widely believed that it can be reduced, or even suppressed, by sheared rotation or a sheared radial electric field. As we have seen, if the edge plasma is collisional (which is the case in MAST), the toroidal rotation and radial electric field should be highest when the neutrals are localized on the inboard side, and the shear should then also be highest. The fact that inboard refueling has the opposite effect in the banana regime, making the plasma rotate less in the countercurrent direction, should further increase the shear if the plasma is in this regime further into the core. On the other hand, if the plasma were in the banana regime all the way out to the last closed flux surface, and if the rotation there were caused by this mechanism, then one would expect larger shear when the neutrals are localized on the outboard rather than the inboard side. In both collisionality regimes the presence of heavy impurity ions promotes countercurrent rotation, so that the inboard-outboard difference is enhanced in the Pfirsch–Schlüter regime and reduced in the banana regime. These results suggest that flexible refueling may enable some degree of external control of the tokamak edge, which is desirable since this region plays such an important role for overall plasma performance.

We close with a remark about the experimental verification of these predictions. Instead of detecting bulk plasma rotation directly, it is usually easier to measure the rotation of velocity of impurity ions, from the Doppler shift of their line radiation. When interpreting such measurements, it must be remembered that the rotation speed of impurities is different from that of the bulk ions, which makes a direct comparison with theory difficult. However, the *difference* in angular rotation frequency between inboard and outboard puffing should be the same for all ion species, being related to the corresponding difference in the radial electric field. Another way of testing the theory would be to measure both the toroidal and the poloidal rotation of some impurity species, and compare the inferred the electric field with the theoretical prediction.

ACKNOWLEDGMENTS

This work was funded by EURATOM under association contracts with Sweden and UK, by the UK Engineering and Physical Sciences Research Council, and by the U.S. Department of Energy under Grant No. DE-FG02-91ER-54109.

- ¹M.N. Rosenbluth, P.H. Rutherford, J.B. Taylor, E.A. Frieman, and L.M. Kovrizhnikh, *Plasma Physics and Controlled Nuclear Fusion Research*, Madison, 1971 (International Atomic Energy Agency, Vienna, 1971), Vol. I, p. 495.
- ²F.L. Hinton and S.K. Wong, *Phys. Fluids* **28**, 3082 (1985).
- ³P.J. Catto, I.B. Bernstein, and M. Tessarotto, *Phys. Fluids* **30**, 2784 (1987).
- ⁴J.W. Connor, S.C. Cowley, R.J. Hastie, and L.R. Pan, *Plasma Phys. Controlled Fusion* **29**, 919 (1987).
- ⁵P.J. Catto, P. Helander, J.W. Connor, and R.D. Hazeltine, *Phys. Plasmas* **5**, 3961 (1998).
- ⁶A.N. Simakov and P.J. Catto, *Phys. Plasmas* **10**, 398 (2003).
- ⁷ASDEX Team, *Nucl. Fusion* **29**, 1959 (1989).
- ⁸Y. Miura, Y. Asahi, K. Hanada *et al.*, in *Fusion Energy 1996*, in *Proceedings of the 16th International Conference on Fusion Energy* (IAEA, Vienna, 1997), Vol. 1, p. 167.
- ⁹L.W. Owen, B.A. Carreras, R. Maingi, P.K. Mioduszewski, T.N. Carlstrom, and R.J. Groebner, *Plasma Phys. Controlled Fusion* **40**, 717 (1998).
- ¹⁰T. Fukuda, T. Takizuka, K. Tsuchiya *et al.*, *Nucl. Fusion* **37**, 1199 (1997); T. Fukuda, T. Takizuka, K. Tsuchiya, Y. Kamada, and N. Asakura, *Plasma Phys. Controlled Fusion* **42**, A289 (2000).
- ¹¹R.L. Boivin, J.A. Goetz, A.E. Hubbard *et al.*, *Phys. Plasmas* **7**, 1919 (2000).
- ¹²P. Gohil, L.R. Bayrol, T.C. Jernigan, K.H. Burrell, and T.N. Carlstrom, *Phys. Rev. Lett.* **86**, 644 (2001).
- ¹³M. Valovic, P.G. Carolan, A.R. Field *et al.*, *Plasma Phys. Controlled Fusion* **44**, A175 (2002).
- ¹⁴A.R. Field, P.G. Carolan, R.J. Akers *et al.*, *Plasma Phys. Controlled Fusion* **44**, A113 (2002).
- ¹⁵R.J. Akers, J.W. Ahn, L.C. Appel *et al.*, *Phys. Plasmas* **9**, 3919 (2002).
- ¹⁶F.L. Hinton and R.D. Hazeltine, *Rev. Mod. Phys.* **48**, 239 (1976).
- ¹⁷S.P. Hirshman and D.J. Sigmar, *Nucl. Fusion* **21**, 1079 (1981).
- ¹⁸T. Fülöp, P. Helander, and P.J. Catto, *Phys. Rev. Lett.* **89**, 225003 (2002).
- ¹⁹A.R. Field, P.G. Carolan, N.J. Conway, P.J. Catto, G.F. Counsell, T. Fülöp, P. Helander, H. Meyer, D. Taylor, M.T. Tournianski, M.J. Walsh, and the MAST team, in *30th European Physical Society Conference on Controlled Fusion and Plasma Physics*, St. Petersburg, 7–11 July 2003 (European Physical Society, Geneva, 2003), p. P-2.148.
- ²⁰P. Helander and D.J. Sigmar, *Collisional Transport in Magnetized Plasmas* (Cambridge University Press, Cambridge, 2002).
- ²¹T. Fülöp, P.J. Catto, and P. Helander, *Phys. Plasmas* **5**, 3969 (1998).
- ²²P. Helander, S.I. Krasheninnikov, and P.J. Catto, *Phys. Plasmas* **1**, 3174 (1994).
- ²³M.B. Tendler and O. Ågren, *Phys. Fluids* **25**, 1037 (1982).
- ²⁴S.P. Hirshman, *Phys. Fluids* **20**, 589 (1977).
- ²⁵R.D. Hazeltine, *Phys. Fluids* **17**, 961 (1974).
- ²⁶This is done by solving Eq. (12.24) of Ref. 20 with a Lorentz collision operator.
- ²⁷M. Taguchi, *Plasma Phys. Controlled Fusion* **30**, 1897 (1988).
- ²⁸P. Helander, *Phys. Plasmas* **8**, 4700 (2001).
- ²⁹A.B. Mikhailovskii and V.S. Tsypin, *Sov. J. Plasma Phys.* **10**, 51 (1984).
- ³⁰P.J. Catto and A.N. Simakov, "A drift ordered short mean free path description for magnetized plasma allowing strong spatial anisotropy," *Phys. Plasmas* (submitted).