Nonlinear neoclassical transport in a rotating impure plasma with large gradients

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The theory of neoclassical transport in an impure toroidal plasma is extended to allow for larger pressure and temperature gradients and faster toroidal rotation than are usually considered. Under these conditions, the density of heavy impurities is not constant on flux surfaces, and the neoclassical transport becomes a nonlinear function of the gradients. Rapid toroidal rotation increases the transport, which can significantly exceed the conventional Pfirsch–Schlüter value if the impurity Mach number is of order unity. In a plasma with steep density or temperature profile, the transport is severely reduced and can even be a nonmonotonic function of the gradients. Finally, in the presence of both rapid toroidal rotation and steep gradients, the transport becomes sensitive to the geometry of the magnetic equilibrium. For instance, in a single-null diverted magnetic field the ion particle flux is typically *inward* if the ion drift is toward the X-point and changes direction if the toroidal field is reversed. [S1070-664X(99)00408-5]

I. INTRODUCTION

It is widely recognized that the conventional theory of neoclassical transport in tokamaks[1,2] is not applicable to regions where the pressure and temperature profiles are very steep, such as the pedestal at the plasma edge. The reason for this lies in the orderings of the theory, and there have been a number of attempts to overcome various aspects of this difficulty.[3–7]

In a recent paper,[8] the neoclassical theory of ion transport in an impure plasma was extended to allow for larger gradients than are usually considered. Specifically, the gradients were allowed to be so large that the friction between the bulk ions and heavy impurity ions could compete with the parallel impurity pressure gradient, as is typically the case in the tokamak edge. The impurity dynamics then becomes nonlinear and the impurity density is not constant on flux surfaces.[9] It was found in Ref. 8 that if the pressure and temperature gradients of the main ion species are steep enough the impurities are pushed toward the inside of each flux surface, which reduces their friction with the bulk ions. Since this is the driving force for the neoclassical particle flux, the latter becomes a nonlinear function of the gradients and is suppressed when the gradients become very large. The total (classical + neoclassical) particle flux was found to be a nonmonotonic function for plasma parameters typical of the tokamak edge, and there is thus scope for a transport bifurcation.

The purpose of the present paper is to include the effects of toroidal plasma rotation in this theory. There are two reasons for why rotation could be expected to be important. First, it is well known that the centrifugal force pushes heavy ions to the outside of the torus, and thus has an effect opposite to that of the friction force.[10] This has been observed in many tokamaks.[11,12] Second, it has been shown that the rotation can affect conventional neoclassical transport (without steep gradients). For a pure plasma this effect was investigated in Refs. 13 and 14, and the modifications of the transport were found to be of the order of

$$M_i^2=\frac{m_i\omega^2R^2}{2T_i}, \qquad (1)$$

which is typically not very large in most experiments. Here m_i is the mass of the bulk ions, T_i their temperature, R the major radius, and ω the angular rotation frequency, so that M_i is the bulk ion Mach number. However, it has recently been pointed out that in a plasma with heavy impurities of mass $m_z > m_i$, the effect of rotation can be larger.[15] The nonuniformity of the impurity density over the flux surface caused by the rotation enhances the neoclassical processes if the *impurity* Mach number $M_z^2=m_z\omega^2R^2/2T_i$ is of order unity, even if $M_i^2 \ll 1$. This effect is particularly large in a collisional plasma, where the rotation can increase the diffusivity well above the conventional Pfirsch–Schlüter value.[16]

As in conventional neoclassical theory, we take the basic expansion parameter to be the poloidal Larmor radius of the bulk ions divided by the radial scale length associated with the density and temperature profiles,

$$\delta=\rho_\theta/L_\perp \ll 1, \qquad (2)$$

which ensures that the ion distribution function is approximately Maxwellian.[1] This ordering can easily be violated in the plasma edge, where the ion orbits communicate directly with the plasma edge, which makes transport theory very difficult.

The plasma is assumed to consist of electrons (e) and hydrogenic ions (i), which are in the collisionless (banana) regime, and of highly charged, collisional (Pfirsch–Schlüter) impurities (z). The impurity Mach number is taken to be of order unity, $M_z = O(1)$, so that the main (H) ion Mach number is small, $M_i^2 = O(1/z) \ll 1$. As in Ref. 8, the parameter

$$\Delta \equiv \delta \hat{\nu}_{ii} z^2 \tag{3}$$

is assumed to be of order unity. Here $z \gg 1$ is the impurity charge number, $\hat{\nu}_{ii} = L_\parallel / \lambda_i$ is the ion collisionality, λ_i is the bulk ion mean-free path, and $L_\parallel$ is the connection length. The ordering $\Delta = O(1)$ is the basic point where the theory of Ref. 8 and this paper differs from conventional neoclassical theory, which assumes that δ is so small that $\Delta \ll 1$. In the tokamak edge, the parameter Δ frequently exceeds unity for typical impurities. The importance of the parameter Δ was pointed out in Ref. 7, where transport theory was developed for a collisional, isothermal plasma in a torus with large aspect ratio and circular cross section. Physically, Δ is an estimate of the ratio between the ion-impurity friction force and the parallel impurity pressure gradient. In conventional neoclassical theory, the impurity density is constant on flux surfaces since there is nothing to oppose the parallel pressure gradient when $\Delta \ll 1$. However, when $\Delta = O(1)$ the impurities can rearrange themselves within a flux surface in response to the friction force.

The impurity concentration in a tokamak is usually such that the frequencies of ion–ion collisions and ion–impurity collisions are comparable. We thus assume $Z_{\rm eff} - 1 \equiv n_z z^2 / n_i = O(1)$, although we shall occasionally relax this ordering to study the case when $Z_{\rm eff} - 1$ is either large or small. As we shall see, this ordering and the assumption $M_z = O(1)$ make the poloidal variation in the electrostatic potential relatively weak, which simplifies the kinetic theory by ruling out electrostatic trapping of the bulk ions and electrons.[17] Orbit effects associated with the impurities[18] are not important since they are assumed to be collisional.

The rest of the paper is organized as follows. Section II presents the kinetic equation for the main ions and its solution in the limits of large impurity concentration and large aspect ratio ($\epsilon = r/R \ll 1$), respectively. Here n_z and n_i are the impurity and main ion densities. In Sec. III the parallel impurity dynamics is analyzed, and an equation is derived for the poloidal impurity distribution over a flux surface. The dynamics of the main ions and the impurity ions are coupled in a complicated way, which makes the neoclassical transport nonlinear. In Sec. IV the neoclassical particle and heat fluxes are calculated, and the conclusions are summarized in Sec. V.

II. KINETIC EQUATION FOR THE MAIN IONS

We begin by noting that the orderings (2) and (3) imply that $1/z = O(\delta^{1/2})$. Therefore, the ion Mach number is of the order $M_i = O(1/z) = O(\delta^{1/2})$, which exceeds the diamagnetic rotation speed $V_{\nabla p} / v_{Ti} = O(\delta)$, where $v_{Ti} = (2T_i/m_i)^{1/2}$. The rotation is thus dominated by the $\mathbf{E}\times\mathbf{B}$ drift, and we need to employ a kinetic equation which allows such a large drift velocity. The drift kinetic equation for each species (a) in a toroidally rotating plasma confined by an axisymmetric magnetic field can be written in the following way:[13,14,19]

$$v_\parallel \nabla_\parallel f_a - e_a v_\parallel \nabla_\parallel \tilde{\Phi} \frac{\partial f_a}{\partial H} - C(f_a) = - v_\parallel f_{a0} \sum_{j=1}^{3} A_{aj} \nabla_\parallel \alpha_{aj},$$

where the lowest-order distribution function is

$$f_{a0} = N_a(\psi) \left(\frac{m_a}{2\pi T_a} \right)^{3/2} \exp(-H/T_a),$$

the thermodynamic ''forces'' are

$$A_{a1} = \frac{N_a'}{N_a} + \frac{T_a'}{T_a}, \quad A_{a2} = \frac{T_a'}{T_a}, \quad A_{a3} = \frac{\omega'}{\omega},$$

and

$$\alpha_{a1} = \frac{m_a}{e_a} \left(\frac{I v_\parallel}{B} + \omega R^2 \right), \quad \alpha_{a2} = \left(\frac{H}{T_a} - \frac{5}{2} \right) \alpha_{a1},$$

$$\alpha_{a3} = \frac{m_a^2 \omega}{2 e_a T_a} \left[\left(\frac{I v_\parallel}{B} + \omega R^2 \right)^2 + \mu \frac{|\nabla \psi|^2}{m_a B} \right].$$

Here $\mu = m_a v_\perp^2 / (2B)$ and $H = m_a v^2/2 + e_a \overline{\Phi} - m_a \omega^2 R^2/2$ are the magnetic moment and lowest-order energy, respectively. The magnetic field is written as $\mathbf{B} = I(\psi) \nabla \varphi + \nabla \varphi \times \nabla \psi$, where φ is the toroidal angle and ψ is the poloidal flux function, and a prime denotes differentiation with respect to ψ. The angular frequency of the rotation is $\omega = -d\overline{\Phi}/d\psi$, where $\overline{\Phi}(\psi)$ is the lowest-order electrostatic potential, and the rotation velocity is thus $\omega R^2 \nabla \varphi$. The velocity $\boldsymbol{v}$ is measured in the rotating frame, and the independent velocity-space variables are H and μ.

Since the poloidal variation of the electrostatic potential is of the order $e\tilde{\Phi}/T \sim n_z z / n_i = O(\delta^{1/2})$ it is appropriate to expand the drift kinetic equation in powers of $\delta^{1/2}$. In $O(\delta^{1/2})$ we then have

$$v_\parallel \nabla_\parallel f_{i1/2} + v_\parallel \frac{e \nabla_\parallel \tilde{\Phi}}{T_i} f_{i0} = C_i^l (f_{i1/2}),$$

where C_i^l is the linearized ion collision operator and in $O(\delta)$ we have

$$v_\parallel \nabla_\parallel f_{i1} - C(f_{i1}) = - \frac{e v_\parallel}{T_i} \nabla_\parallel \tilde{\Phi} f_{i1/2} - v_\parallel f_{i0} \sum_{j=1}^{3} A_j \nabla_\parallel \alpha_j.$$

The solution to these equations is conveniently expressed as

$$f_i = f_{i0} - \frac{e\tilde{\Phi}}{T_i} f_{i0} + \frac{1}{2} \left(\frac{e\tilde{\Phi}}{T_i} \right)^2 f_{i0} - \sum_{j=1}^{3} A_j \alpha_j f_{i0} + h_i, \tag{4}$$

where the function h_i satisfies

$$v_\parallel \nabla_\parallel h_i = C_i^l (f_{i1}) \tag{5}$$

and $\sigma = v_\parallel / |v_\parallel|$. In the low-collisionality (banana) regime, h_i is a function only of constants of motion, i.e., $h_i = h_i(H, \mu, \psi, \sigma)$, and vanishes for trapped particles.

It follows from the assumption of low Mach number, $M_i^2 = O(1/z)$, that $f_{i0}' \gg 2 M_i^2 (\omega'/\omega) f_{i0}$, unless the radial electric field is very strongly sheared. (We expect ω to vary on the same length scale $L_\perp$ as the pressure, which rules out such strong shear.) The main ion distribution function (4) can thus be rewritten as

$$f_i = f_{i0}\exp\left(-\frac{e\tilde{\Phi}}{T_i}+M_i^2\right)-\frac{Iv_\parallel}{\Omega_i}\frac{\partial f_{i0}}{\partial\psi}+h_i(H,\mu,\psi,\sigma), \tag{6}$$

where $\Omega_i = eB/m_i$ is the ion cyclotron frequency. A similar result holds for the electrons, and the densities of these species are thus

$$n_i(\psi,\theta)=n_{i0}\left(1-\frac{e\tilde{\Phi}}{T_i}+M_i^2+O(\delta)\right), \tag{7}$$

$$n_e(\psi,\theta)=n_{e0}\left(1+\frac{e\tilde{\Phi}}{T_e}+O(\delta)\right), \tag{8}$$

where we have anticipated that h_i is odd in σ and therefore carries no density. Note that the Mach number varies over the flux surface.

Using the expression (6) for the perturbed ion distribution function we proceed to calculate the parallel friction force between the H ions and the impurities

$$R_{zi\parallel}=-\int m_i v_\parallel \nu_{iz}(v)\left(\mathcal{L}(f_i-f_{i0})+\frac{m_i v_\parallel}{T_i}V_{z\parallel}f_{i0}\right)d^3v, \tag{9}$$

where ion-impurity collisions are described by the operator

$$C_{iz}^l \simeq \nu_{iz}\left(\mathcal{L}+\frac{m_i v_\parallel}{T_i}V_{z\parallel}f_{i0}\right)$$

since the mass ratio is large, $m_z/m_i \gg 1$. Here the Lorentz scattering operator is defined as

$$\mathcal{L}=\frac{2v_\parallel}{v^2 B}\frac{\partial}{\partial\lambda}\lambda v_\parallel\frac{\partial}{\partial\lambda},$$

with $\lambda = v_\perp^2/(Bv^2)$, and the ion–impurity collision frequency is equal to $\nu_{iz}=3\pi^{1/2}/(4\tau_{iz}x^3)$, with $x=v/v_{Ti}$. The ion–impurity collision time is

$$\tau_{iz}=\frac{3(2\pi)^{3/2}\epsilon_0^2\sqrt{m_i}T_i^{3/2}}{n_z z^2 e^4\ln\Lambda},$$

and the parallel impurity flow velocity is given by[8]

$$V_{z\parallel}=-\frac{I}{B}\frac{d\Phi_0}{d\psi}+\frac{K_z(\psi)B}{n_z},$$

where $K_z(\psi)$ is proportional to the poloidal velocity. The velocity space element is equal to $d^3v = \Sigma_\sigma 2\pi B/(m_i^2|v_\parallel|)dHd\mu$. Using these relations to evaluate the integral in Eq. (9) and recalling the ordering $M_i^2 \ll 1$, we obtain the friction force

$$R_{zi\parallel}=-\frac{p_i I}{\Omega_i\tau_{iz}}\left(\frac{p_i'}{p_i}-\frac{3}{2}\frac{T_i'}{T_i}\right)+\frac{m_i n_i}{\tau_{iz}}\left(u-\frac{K_z}{n_z}\right)B, \tag{10}$$

where

$$u=\frac{\tau_{iz}}{n_i B}\int v_\parallel \nu_{iz} h_i d^3v \tag{11}$$

is a flux function. This result is the same as in the nonrotating case.[8] The first term in Eq. (10) represents the friction force from the diamagnetic flow associated with the guiding-center orbits of the bulk ions. This force is proportional to the orbit width and is thus *inversely proportional* to B. The last term is the friction associated with the parallel flow of impurities $n_z V_{\parallel z}=K_z B$, which, by particle conservation, is inversely proportional to the cross section of the flux tube along which the impurities flow. This term in the friction force is therefore *proportional* to the magnetic field strength B. Since the terms thus scale differently with B, the friction force varies over the flux surface and, as we shall see, can cause a poloidal rearrangement of the impurities.

It remains to solve Eq. (5) to determine the unknown quantity u in the friction force (10). We shall do this in two different limiting cases: first for a high level of impurities, $Z_{\rm eff}\gg 1$, in arbitrary flux-surface geometry, and second for large aspect ratio with arbitrary $Z_{\rm eff}$.

A. Solution in the limit $Z_{\rm eff}\gg 1$

In order to solve Eq. (5) in the passing region, we follow the conventional procedure of multiplying the equation by $B/v_\parallel$ and taking the flux-surface average $\langle\cdots\rangle$,

$$\left\langle\frac{B}{v_\parallel}C_i^l\left(h-\frac{Iv_\parallel}{\Omega_i}\frac{\partial f_{i0}}{\partial\psi}\right)\right\rangle=0. \tag{12}$$

If $Z_{\rm eff}\gg 1$, ion–impurity collisions dominate over ion–ion collisions which can therefore be neglected. Equation (12) then gives

$$\frac{\partial h_i}{\partial\lambda}=\frac{x^2 f_{i0}}{\langle n v_\parallel\rangle}\left(-\frac{IT_i}{e}\frac{\partial\ln f_{i0}}{\partial\psi}+I\frac{d\Phi}{d\psi}-K_z\frac{\langle B^2\rangle}{\langle n_z\rangle}\right)H(\lambda_c-\lambda), \tag{13}$$

where we have introduced the normalized impurity density $n\equiv n_z/\langle n_z\rangle$, and where H is the Heaviside step function, which ensures that h_i vanishes in the trapped region $\lambda>\lambda_c \equiv 1/B_{\max}$. Using this result in Eq. (11) we can calculate

$$\frac{u}{f_c}=-\frac{T_i}{e\langle B^2\rangle L_\perp}+\frac{K_z}{\langle n_z\rangle}, \tag{14}$$

where the radial scale length is defined by

$$L_\perp^{-1}=-I\left(\frac{p_i'}{p_i}-\frac{3}{2}\frac{T_i'}{T_i}\right),$$

and the ''effective fraction'' of circulating particles is

$$f_c\equiv\frac{3\langle B^2\rangle}{4}\int_0^{\lambda_c}\frac{\lambda\, d\lambda}{\langle n\sqrt{1-\lambda B}\rangle}, \tag{15}$$

which differs from the definition of Hirshman and Sigmar[2] if the impurity density varies over the flux surface, i.e., if $n \neq 1$. In Eq. (14), the remaining unknown quantity, K_z, which governs the poloidal impurity rotation, will be determined from the parallel impurity momentum equation in the next section.

B. Solution in the limit $\epsilon \ll 1$

We now turn to the case of arbitrary impurity concentration, $Z_{\text{eff}}-1=O(1)$, but instead simplify the problem by assuming that the flux surface has large aspect ratio. Both ion–ion and ion–impurity collisions must now be retained in Eq. (12), which, however can be made tractable by describing ion–ion collisions by the Kovrizhnikh operator,[20]

$$C_{ii}^{l}(f_i) \simeq \nu_{ii}\left[\mathcal{L}(f_i)+\frac{m_i v_\parallel}{T_i}\hat{p} f_{i0}\right],$$

where the constant $\hat{p}$ is determined from momentum conservation

$$\int m_i v_\parallel C_{ii}(f_{i1}) d^3v$$

$$=\int m_i v_\parallel \nu_{ii}\left(\frac{I v_\parallel}{\Omega_i}\frac{\partial f_{i0}}{\partial \psi}-h_i+\frac{m_i v_\parallel}{T_i}\hat{p} f_{i0}\right) d^3 v=0.$$

In these equations, the ion–ion collision frequency is

$$\nu_{ii}=\frac{n_i e^4 \ln\Lambda}{4\pi m_i^2 \epsilon_0^2 v_{Ti}^3}\frac{\phi(x)-G(x)}{x^3},$$

where $G(x)$ is the Chandrasekhar function, which is defined by

$$G(x)\equiv\frac{\phi(x)-x\phi'(x)}{2x^2},$$

$$\phi(x)\equiv\frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2}dy.$$

Equation (12) now gives

$$\frac{\partial h_i}{\partial \lambda}=-\frac{x^2 f_{i0}}{\langle \nu_i v_\parallel\rangle}\left(\frac{IT_i}{e}\langle \nu_i\rangle\frac{\partial \ln f_{i0}}{\partial \psi}+\langle B(\nu_{ii}\hat{p}+\nu_{iz}V_{z\parallel})\rangle\right), \quad (16)$$

where $\nu_i=\nu_{ii}+\nu_{iz}$ and $\hat{p}$ is determined by

$$\langle B\hat{p}\rangle=\langle BV_{z\parallel}\rangle-\frac{f_t I T_i}{e}\left\{\frac{\nu_{ii}(\nu_{iz}+f_t\nu_{ii})}{\nu_i}\right\}^{-1}$$
$$\times\left\{\nu_{ii}\left(\frac{p_i'}{p_i}+\left(x^2-\frac{5}{2}\right)\frac{T_i'}{T_i}+\frac{eK_z}{I}\left\langle\frac{B^2}{p_z}\right\rangle\right)\right\}. \quad (17)$$

Here we have assumed that the aspect ratio is large, $\epsilon \ll 1$, so that the variation in the impurity density over the flux surface is $O(\epsilon)$ and the collision frequency ν_i does not vary much over the flux-surface. Solving the kinetic Eq. (16) and calculating u, which was defined in Eq. (11), then yields after some algebra

$$\frac{u}{f_c}=-\frac{T_i}{e\langle B^2\rangle L_\perp}+K_z\left\langle\frac{1}{n_z}\right\rangle-\frac{f_t I T_i}{e\langle B^2\rangle}\left\{\tau_{iz}\frac{\nu_{ii}\nu_{iz}}{\nu_i}\right\}$$
$$\times\left\{\frac{\nu_{ii}(\nu_{iz}+f_t\nu_{ii})}{\nu_i}\right\}^{-1}\left\{\nu_{ii}\left(\frac{p_i'}{p_i}+\left(x^2-\frac{5}{2}\right)\frac{T_i'}{T_i}\right.\right.$$
$$\left.\left.+\frac{eK_z}{I}\left\langle\frac{B^2}{p_z}\right\rangle\right)\right\}, \quad (18)$$

where $f_t=1-f_c$ is the effective number of trapped particles, and

$$\{\cdots\}\equiv\frac{8}{3\sqrt{\pi}}\int_0^\infty(\cdots)x^4\exp(-x^2)dx$$

is a conveniently defined average over velocity space. Since the impurity density is nearly constant over the flux surface, $n=1+O(\epsilon)$, the definition (15) of f_c now coincides with that of Hirshman and Sigmar,

$$f_c=\frac{3\langle B^2\rangle}{4}\int_0^{\lambda_c}\frac{\lambda\, d\lambda}{\sqrt{1-\lambda B}}. \quad (19)$$

Note that in the limit $Z_{\text{eff}}-1\ll 1$ (trace impurities) we can let $\nu_{iz}\rightarrow 0$ and the expression for u becomes independent of the impurities,

$$\frac{u}{f_c}=-\frac{I}{e\langle B^2\rangle}\frac{\{\nu_{ii}(x^2-1)\}}{\{\nu_{ii}\}}\frac{\partial T_i}{\partial\psi}=-c_0\frac{I}{e\langle B^2\rangle}\frac{\partial T_i}{\partial \psi}, \quad (20)$$

where $c_0=\{\nu_{ii}(x^2-1)\}/\{\nu_{ii}\}\simeq 0.33$. In the opposite limit ($Z_{\text{eff}}\gg 1$) we can let $\nu_{ii}\rightarrow 0$ which makes the last term in Eq. (18) vanish and we recover Eq. (14) with an $O(\epsilon)$ error.

III. PARALLEL IMPURITY DYNAMICS

The kinetics of the bulk ions we have just analyzed only differs from conventional neoclassical theory because the ion-impurity collision frequency varies over the flux surface. In addition, the poloidal rotation of the impurities, K_z, will turn out to be different, but this did not affect any of the results in the previous section.

The impurity dynamics is more complicated and becomes nonlinear when the bulk plasma gradients are large. The parallel momentum equation for the impurities, including the centrifugal force, is

$$m_z n_z \mathbf{b}\cdot(\mathbf{V}_z\cdot\nabla)\mathbf{V}_z=-zn_z e\nabla_\parallel\tilde{\Phi}-T_i\nabla_\parallel n_z+R_{zi\parallel}, \quad (21)$$

where $\mathbf{b}=\mathbf{B}/B$ and we have neglected the parallel viscosity of the impurities since it was shown in Ref. 8 to be smaller than the pressure gradient if $\delta/z\hat{\nu}_{ii}\ll 1$, which is usually the case in the tokamak edge. As also shown in that paper, the impurity temperature is then equilibrated with the bulk ion temperature and is therefore constant over the flux surface. (The bulk ions are collisionless and their temperature is therefore necessarily a flux function. We neglect the heat transfer between impurities and electrons.[21]) Since $\mathbf{V}_z \simeq \omega R\hat{\varphi}$, the centrifugal force is equal to $-m_z\mathbf{b}\cdot(\mathbf{V}_z\cdot\nabla)\mathbf{V}_z = m_z\omega^2 R(\mathbf{b}\cdot\mathbf{R})$. The electrostatic potential $\tilde{\Phi}$ can be obtained from the quasineutrality condition $zn_z=n_e-n_i$ leading to

$$\frac{ze\nabla_\parallel\tilde{\Phi}}{T_i}=\frac{T_0}{2T_i n_0}\nabla_\parallel(z^2 n_z+zn_{i0}M_i^2), \quad (22)$$

where M_i is the bulk ion Mach number (1), $2n_0/T_0 \equiv n_{e0}/T_e+n_{i0}/T_i$, and we have used Eqs. (7) and (8). The parallel momentum equation now becomes

$$(1+\alpha n)\nabla_{\parallel} n = n\nabla_{\parallel} M^2 + \frac{R_{zi\parallel}}{\langle n_z\rangle T_i}, \tag{23}$$

where the friction force was given in Eq. (10), and we have introduced $\alpha \equiv \langle n_z\rangle z^2 T_0/2n_0 T_i$ and a modified impurity Mach number

$$M^2 = \frac{m_z \omega^2 R^2}{2T_i}\left(1 - \frac{z m_i}{m_z}\frac{T_e}{T_e+T_i}\right) = O(1).$$

To rewrite Eq. (23) in dimensionless form we further introduce the normalized magnetic field strength $b \equiv B/\langle B^2\rangle^{1/2}$, the parameters

$$g = \frac{m_i n_i}{eL_\perp \tau_{iz} n_z \langle \mathbf{B}\cdot\nabla\theta\rangle}, \quad \gamma = \frac{eL_\perp \langle B^2\rangle u}{T_i}, \tag{24}$$

and a modified poloidal angle coordinate ϑ defined as

$$\frac{d\vartheta}{d\theta} \equiv \frac{\langle \mathbf{B}\cdot\nabla\theta\rangle}{\mathbf{B}\cdot\nabla\theta},$$

which makes the flux-surface average equivalent to an average over ϑ. Equation (23) now becomes

$$(1+\alpha n)\frac{\partial n}{\partial\vartheta} = g\left(n + \gamma\left(n - \frac{K_z}{\langle n_z\rangle u}\right)b^2\right) + \frac{\partial M^2}{\partial\vartheta} n. \tag{25}$$

Integrating Eq. (25) over ϑ yields a solubility constraint which can be used to determine the poloidal impurity rotation,

$$K_z = \langle n_z\rangle u\left(\langle nb^2\rangle + \frac{1}{\gamma} + \frac{1}{\gamma g}\left\langle n\frac{\partial M^2}{\partial\vartheta}\right\rangle\right), \tag{26}$$

and Eq. (25) governing the distribution of the impurities on the flux surface becomes

$$(1+\alpha n)\frac{\partial n}{\partial\vartheta} = g(n - b^2 + \gamma(n - \langle nb^2\rangle)b^2) + \frac{\partial M^2}{\partial\vartheta} n - \left\langle n\frac{\partial M^2}{\partial\vartheta}\right\rangle b^2. \tag{27}$$

This equation plays a central role in the present theory. The two most important control parameters in it are g, which measures the steepness of the bulk ion density and temperature profiles, and the impurity Mach number M associated with the toroidal rotation. The parameter g is of the same order as Δ, which was defined in Eq. (3). In conventional neoclassical theory g is thus assumed to be small, which implies that the friction force is smaller than the parallel pressure gradient. It was shown in Ref. 8 that the impurities are pushed to the inboard side of the torus when g becomes large, and the neoclassical transport then becomes a strongly nonlinear function of the gradients. As discussed in many papers,[7,10,15] impurities are pushed to the outside of the torus by the centrifugal force when their Mach number M is large.

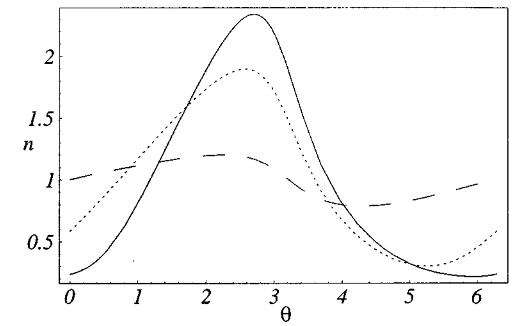

FIG. 1. Normalized impurity density $n = n_z/\langle n_z\rangle$ as a function of the poloidal angle θ for a typical START discharge (No. 35096). The impurity Mach number at the magnetic axis is $M_0^2 = 1$, the impurity strength is $\alpha = 5$, and the normalized gradients are $g = 1$ (dashed line), $g = 5$ (dotted line), and $g = 10$ (solid line). The impurity distribution is calculated by solving Eq. (27) using Eq. (28). Note the up–down asymmetry for $g = 1$, and the accumulation of the impurities on the inside of the flux–surface for steeper gradients.

Knowing the poloidal impurity rotation, we can now calculate u in the limits of large $Z_{\rm eff}$ and large aspect ratio, respectively. In the former limit, i.e., $\alpha \sim (Z_{\rm eff}-1)/2 \gg 1$, we use (26) in (14) and obtain

$$u = \frac{\hat{\mu}}{f_c^{-1} - \langle nb^2\rangle}, \tag{28}$$

where $\hat{\mu} = p_z \tau_{iz} \langle \mathbf{B}\cdot\nabla\theta\rangle \langle n\partial M^2/\partial\theta\rangle/(m_i n_i \langle B^2\rangle)$, while in the limit of large aspect ratio but arbitrary impurity density, $\alpha = O(1)$, $\epsilon \ll 1$, we find from Eqs. (18) and (26),

$$u = \frac{1}{f_c^{-1} - (\beta+1)}\left(\hat{\mu}(1-\beta) - \beta c_0 \frac{IT_i'}{e\langle B^2\rangle}\right), \tag{29}$$

with

$$\beta = f_t\left\{\frac{\tau_{iz}\nu_{ii}\nu_{iz}}{\nu_i}\right\}\left\{\frac{\nu_{ii}(\nu_{iz} + f_t\nu_{ii})}{\nu_i}\right\}^{-1}\{\nu_{ii}\}.$$

By using these results, Eq. (27) can be solved numerically for arbitrary gradients, in the two limiting cases $\alpha \gg 1$ and $\alpha = O(1), \epsilon \ll 1$. Figure 1 shows the impurity density variation over a flux surface close to the edge in a typical discharge (No. 35096) in the Small Tight Aspect Ratio Tokamak (START) at Culham.[22] The magnetic reconstruction is shown in Fig. 2. When the gradients are weak ($g = 1$), the density is mostly up–down asymmetric, but at larger gradients the impurities are pushed to the inboard side of the flux surface. The impurity density then becomes an order of magnitude larger on the inside than on the outside of the flux surface, which should be an experimentally observable effect.

IV. NEOCLASSICAL TRANSPORT

The two previous sections were concerned with the parallel dynamics of the bulk ions and impurities, respectively, and these were found to be closely coupled to each other. The poloidal rotation and poloidal distribution of impurities are controlled by the ion–impurity friction force through the

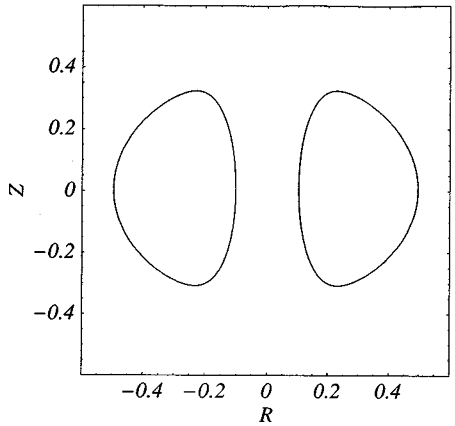

FIG. 2. A magnetically reconstructed flux surface close to the edge in START (discharge No. 35096).

parameters g and γ in Eq. (27). The magnitude of the latter is determined by the bulk ion distribution function, which in turn depends on the poloidal distribution and rotation of the impurities. Thus is the ion-impurity system self-consistently coupled.

In this section, we turn our attention to the neoclassical transport across the magnetic field. Because of the nontrivial parallel dynamics, the radial transport is nonlinear and therefore exhibits a number of unusual features. The radial particle flux is obtained in a conventional way by taking the toroidal component of the momentum equation,

$$m_i n_i (\mathbf{V}_i \cdot \nabla)\mathbf{V}_i = n_i e(\mathbf{E}+\mathbf{V}_i \times \mathbf{B}) - \nabla p_i - \nabla \cdot \boldsymbol{\pi}_i + \mathbf{R}_{iz},$$

where $\mathbf{R}_{iz} = -\mathbf{R}_{zi}$. Since $R^2 \nabla \varphi \cdot (\mathbf{V}_i \times \mathbf{B}) = \mathbf{V}_i \cdot \nabla \psi$, the flux becomes

$$e\boldsymbol{\Gamma}_i \cdot \nabla \psi = R^2 \nabla \varphi \cdot \mathbf{R}_{iz},$$

where we have neglected the induced electric field E_φ and the viscosity. The flux associated with the parallel component of the friction force $\mathbf{R}_{iz}$ is the neoclassical flux,

$$\boldsymbol{\Gamma}_i^{\text{neo}} \cdot \nabla \psi = \frac{I R_{iz\parallel}}{eB} = \frac{I}{eB} \int m_i v_\parallel C_i(f_i) d^3 v, \tag{30}$$

and the corresponding flux associated with the perpendicular component of the friction is the classical flux, which is typically smaller than the neoclassical one. Using the friction force $R_{zi\parallel}$ from Eqs. (10) and (26) gives the average neoclassical particle flux across a flux surface

$$\langle \boldsymbol{\Gamma}_i^{\text{neo}} \cdot \nabla \psi \rangle = \frac{I \langle p_z \rangle \langle \mathbf{B} \cdot \nabla \theta \rangle}{e \langle B^2 \rangle} \left\{ g \left[\left\langle \frac{n}{b^2} \right\rangle - 1 + \gamma (1 - \langle n b^2 \rangle) \right] - \left\langle n \frac{\partial M^2}{\partial \vartheta} \right\rangle \right\}. \tag{31}$$

We can eliminate the Mach number from this expression by noting that dividing Eq. (27) by n and taking the flux surface average gives

$$\left\langle n \frac{\partial M^2}{\partial \vartheta} \right\rangle = g \left[(1+\gamma) \left\langle \frac{b^2}{n} \right\rangle^{-1} - 1 - \gamma \langle n b^2 \rangle \right]. \tag{32}$$

The neoclassical particle flux can thus be written as

$$\langle \boldsymbol{\Gamma}_i^{\text{neo}} \cdot \nabla \psi \rangle = g \frac{I \langle p_z \rangle \langle \mathbf{B} \cdot \nabla \theta \rangle}{e \langle B^2 \rangle} \left[\left\langle \frac{n}{b^2} - 1 \right\rangle + (1+\gamma) \left(1 - \left\langle \frac{b^2}{n} \right\rangle^{-1} \right) \right]. \tag{33}$$

It follows from this relation that the transport is proportional to the pressure and temperature gradients when these are small, $g \ll 1$, since the lowest-order impurity distribution $n(\vartheta)$ is then determined by Eq. (27) with $g=0$. When the gradients are larger, $g = O(1)$, the friction force causes impurity redistribution so that n depends on g and the flux is no longer linearly proportional to n. Another useful way of writing the flux is obtained by using Eq. (27),

$$\langle \boldsymbol{\Gamma}_i^{\text{neo}} \cdot \nabla \psi \rangle = \frac{I \langle p_z \rangle \langle \mathbf{B} \cdot \nabla \theta \rangle}{e \langle B^2 \rangle} \times \left\langle b^{-2} \left[(1+\alpha n) \frac{\partial n}{\partial \vartheta} - n \frac{\partial M^2}{\partial \vartheta} \right] \right\rangle. \tag{34}$$

The heat flux can be obtained in a similar manner by calculating the ''heat friction'' $H_{iz\parallel}$,

$$\langle \mathbf{q}_i^{\text{neo}} \cdot \nabla \psi \rangle = \left\langle \frac{I H_{iz\parallel}}{eB} \right\rangle = \left\langle \frac{I T_i}{eB} \int m_i v_\parallel \left(x^2 - \frac{5}{2} \right) C_i(f_i) d^3 v \right\rangle.$$

Toroidal rotation affects the transport in two ways: directly through the last terms in Eqs. (31), (33), and (34), and indirectly by changing the distribution of impurities, n, as well as the friction associated with the poloidal rotation, γ. We shall now use the knowledge about these quantities gained in the previous section to evaluate the neoclassical transport in various limits.

A. Large aspect ratio

We begin with the simplest limit, that of a plasma with small inverse aspect ratio, $\epsilon \ll 1$, and circular cross section. This case is analytically tractable for arbitrary gradients g, Mach numbers M, and impurity fractions α since the magnetic field, the impurity density, and the Mach number can be expanded in ϵ as

$$b^2 = 1 - 2\epsilon \cos \theta + O(\epsilon^2),$$

$$n = 1 + n_c \cos \theta + n_s \sin \theta + O(\epsilon^2),$$

$$M^2 = M_0^2 (1 + 2\epsilon \cos \theta) + O(\epsilon^2).$$

The solution of Eq. (27) is then found to be

$$n_s = 2\epsilon g \frac{(1+\alpha) + (1+\gamma) M_0^2}{(1+\alpha)^2 + (1+\gamma)^2 g^2},$$

$$n_c = 2\epsilon \frac{(1+\alpha) M_0^2 - (1+\gamma) g^2}{(1+\alpha)^2 + (1+\gamma)^2 g^2}.$$

It can be noted from the expression for n_c that toroidal rotation causes the impurities to accumulate on the outside of the flux surface, whereas large gradients, $g \gg 1$, have the oppo-

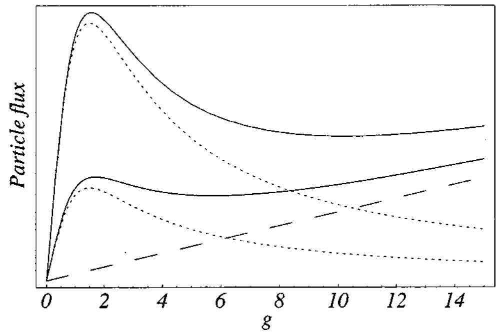

FIG. 3. Ion particle fluxes versus normalized gradient g in a large-aspect-ratio tokamak with circular cross section, $\epsilon \ll 1$, $\alpha = 0.5$, $q = 3$. The dashed line is the classical flux, the dotted lines are neoclassical fluxes, and the solid lines represent the sum of classical and neoclassical fluxes. The lower pair of dotted and solid lines are for vanishing toroidal rotation, $M_0^2 = 0$, and the upper pair for impurity Mach number $M_0^2 = 1$. Note that the neoclassical fluxes are enhanced by finite rotation and suppressed by large gradients.

site effect of pushing the impurities to the inside. The radial particle flux (34) is proportional to the up–down asymmetry of the impurity density,

$$\langle \mathbf{\Gamma}^{\rm neo} \cdot \nabla \psi \rangle = \frac{I \langle p_z \rangle \langle \mathbf{B} \cdot \nabla \theta \rangle}{e \langle B^2 \rangle} (1 + \alpha + M_0^2) \epsilon n_s . \quad (35)$$

Adding the classical particle flux[8] (which is not much affected by the impurity redistribution) to this expression gives the total ion particle transport

$$\langle \mathbf{\Gamma}_i^{\rm cl} \cdot \nabla \psi \rangle + \langle \mathbf{\Gamma}_i^{\rm neo} \cdot \nabla \psi \rangle = \frac{\epsilon^2 p_z}{q^3 e} \left[1 + \left(1 + \frac{M_0^2}{1+\alpha} \right) \left(1 + \frac{1+\gamma}{1+\alpha} M_0^2 \right) \times \frac{2q^2}{1 + (1+\gamma)^2/(1+\alpha)^2 g^2} \right] g, \quad (36)$$

where $q = rB/RB_\theta$ is the safety factor. The first term is the classical flux and the second term is the neoclassical flux. The latter exceeds the former by the Pfirsch–Schlüter factor $2q^2$ when the gradients and the rotation are weak, $g \ll 1$ and $M_0^2 \ll 1$. When either g or M_0 is not small, new and potentially important effects emerge.

First, if the pressure or temperature gradient becomes sufficiently steep ($g \gg 1$) the neoclassical flux is suppressed since the denominator in the second term of Eq. (36) depends quadratically on g. (The dependence of γ on g is typically quite weak and is unimportant in this context.) Classical transport then dominates, and the total flux is a nonmonotonic function of the gradients. That this is the case in a nonrotating plasma was found in Ref. 8; here we note that it is not affected by toroidal rotation. Figure 3 shows the particle flux as a function of g. Conventional transport theory only covers the lower left corner of this figure.

The second conclusion to draw from Eq. (36) is that if the gradients are weak but the rotation is significant, i.e., if $g \ll 1$ and $M_0 = O(1)$, the neoclassical flux is increased by a factor

$$\Lambda = \left(1 + \frac{M_0^2}{1+\alpha} \right) \left(1 + \frac{1+\gamma}{1+\alpha} M_0^2 \right) \quad (37)$$

over the usual Pfirsch–Schlüter result.[23] The diffusion coefficient thus becomes $D = (1 + 2\Lambda q^2) D_{\rm cl}$, where $D_{\rm cl} = T_i/(m_i \Omega_i^2 \tau_{iz})$ is the classical diffusion coefficient and $2q^2 D_{\rm cl}$ the Pfirsch–Schlüter diffusion coefficient. The enhancement factor Λ can be very large if the impurity Mach number M_0 exceeds unity, as is frequently the case experimentally for heavy impurities. This effect is much larger than that previously reported for impurities in the banana regime[15] and is comparable to that when both species are in the collisional regime, which was recently calculated in Ref. 16.

B. High level of impurities, $\alpha \gg 1$

For a simple explanation why the flux is enhanced by toroidal rotation, it is useful to consider the limit of a very impure plasma, $\alpha \gg 1$, but to keep the aspect ratio arbitrary. In this limit, the friction force is given by Eqs. (10) and (14),

$$R_{zi\|} = -\frac{p_i I}{\Omega_i \tau_{iz}} \left(\frac{p_i'}{p_i} - \frac{3}{2} \frac{T_i'}{T_i} \right) \left(1 - \frac{B^2}{\langle B^2 \rangle} \right) + \frac{m_i n_i K_z B}{\tau_{iz}} \left(\frac{1}{\langle n_z \rangle} - \frac{1}{n_z} \right).$$

In conventional neoclassical theory, where the gradients are weak and the rotation is slow, the impurity density is constant over the flux surface, $n_z \simeq \langle n_z \rangle$, so that the second term in $R_{zi\|}$ disappears and the neoclassical particle flux (30) becomes

$$\mathbf{\Gamma}_i \cdot \nabla \psi = -\frac{p_i m_i I^2}{e^2 \langle B^2 \rangle \tau_{iz}} \left(\frac{p_i'}{p_i} - \frac{3}{2} \frac{T_i'}{T_i} \right) (b^{-2} - 1).$$

This flux is inward on the inboard side (where $b > 1$) and outward on the outboard side of the flux surface. At large aspect ratio these fluxes nearly cancel on a flux–surface average since $\langle b^{-2} - 1 \rangle = O(\epsilon^2)$. However, if the impurity density varies significantly over the flux surface, then the collision time $\tau_{iz} \propto n^{-1}$ has a poloidal variation and must be included inside the flux-surface average. This disturbs the balance between the inboard and outboard fluxes, and can increase the net transport significantly since then $\langle n(b^{-2} - 1) \rangle = O(\epsilon)$.

In order to derive the general expression for the neoclassical particle flux in the limit $\alpha \gg 1$, we use the relation (28) in the definition (24) for γ,

$$\gamma = \frac{1}{g} \frac{\langle n \partial M^2 / \partial \vartheta \rangle}{(f_c^{-1} - \langle n b^2 \rangle)}, \quad (38)$$

and insert this result in Eq. (33) to find

$$\langle \mathbf{\Gamma}^{\rm neo} \cdot \nabla \psi \rangle = -\frac{p_i I^2 \langle \tau_{iz}^{-1} \rangle}{m_i \langle \Omega_i^2 \rangle} \left(\frac{p_i'}{p_i} - \frac{3}{2} \frac{T_i'}{T_i} \right) \times \left(\left\langle \frac{n}{b^2} - 1 \right\rangle + f_t \frac{\langle b^2/n \rangle - 1}{\langle b^2/n \rangle - f_c} \right). \quad (39)$$

The heat flux can be calculated in a similar way by computing the heat friction,

$$H_{iz\|}=T_i\int\left(x^2-\frac{5}{2}\right)m_i v_\| \nu_{iz}\times\left(\frac{Iv_\|}{\Omega_i}\frac{\partial f_{i0}}{\partial\psi}-h_i+\frac{m_i v_\|}{T_i}V_{z\|}f_{i0}\right)d^3v,$$

using Eqs. (13), (26), (32), and (38), with the result

$$\langle\mathbf{q}^{\rm neo}\cdot\nabla\psi\rangle=-\tfrac{3}{2}\langle\boldsymbol{\Gamma}^{\rm neo}\cdot\nabla\psi\rangle T_i-\frac{p_iT_iI^2\langle\tau_{iz}^{-1}\rangle}{m_i\langle\Omega_i^2\rangle}\left(\left\langle\frac{n}{b^2}-1\right\rangle+f_t\right)\frac{T_i'}{T_i}. \quad (40)$$

In the absence of large gradients and toroidal rotation, $g\ll 1$, $M\ll 1$, the particle flux (39) and the heat flux (40) agree with the results of Ref. 23. The quantity $\langle n/b^2-1\rangle$ in these expressions then reduces to the ordinary Pfirsch–Schlüter factor $\langle b^{-2}-1\rangle$, and the effective fraction of trapped particles, $f_t=1-f_c$, which is defined by Eq. (15), coincides with its conventional counterpart (19). As pointed out in Ref. 23, in the mixed collisionality regime (banana regime ions + high-z fluid impurities) we are considering, the particle flux then scales as the Pfirsch–Schlüter value for a collisional plasma while the heat flux (40) scales like that in the low-collisionality (banana) regime.

We have already noted that toroidal rotation with $M=O(1)$ substantially increases the value of the factor $\langle n/b^2-1\rangle$. The rotation has a similar, but less drastic, effect on the effective trapped-particle fraction f_t. For instance, in a circular tokamak with large aspect ratio $\langle b^{-2}-1\rangle\simeq 2\epsilon^2$ and $f_t\simeq 1.46\epsilon^{1/2}$. In the limit of very fast toroidal rotation, $M\gg 1$, where all impurities are concentrated near the outer midplane, $\langle n/b^2-1\rangle\simeq 2\epsilon$ and $f_t\simeq 3(\epsilon/2)^{1/2}$, see Ref. 15. Thus, the Pfirsch–Schlüter factor is increased by a factor ϵ^{-1} and the effective trapped-particle fraction by 45%. Note also that there is an additional positive particle flux from the last term in Eq. (39).

Figure 4 shows the neoclassical particle and heat fluxes vs the gradients g for the START discharge (No. 35096) we discussed earlier. The density gradient has been chosen to be twice as large as the temperature gradient, as is typical in experiments. At small gradients, $g=O(1)$, the first term in the heat flux (40) then dominates so that the net flux is inward. (The energy flux $Q_i=q_i+5T_i\Gamma_i/2$ is of course outward.) However, as the particle flux is suppressed at very large gradients, $g\gg 1$, the second term becomes more important and the heat flux is outward. Note that the heat flux is smaller than it would have been in conventional neoclassical theory, which is obtained by setting $n=1$ in Eq. (40).

C. Large gradients

Finally, we consider particle transport in the limit when the pressure or temperature gradient is so large that $g\gg 1$, while the aspect ratio and impurity concentration are arbitrary. Expanding the solution of Eq. (27) in g^{-1},

$$n=n_0+n_1+O(g^{-2}),$$

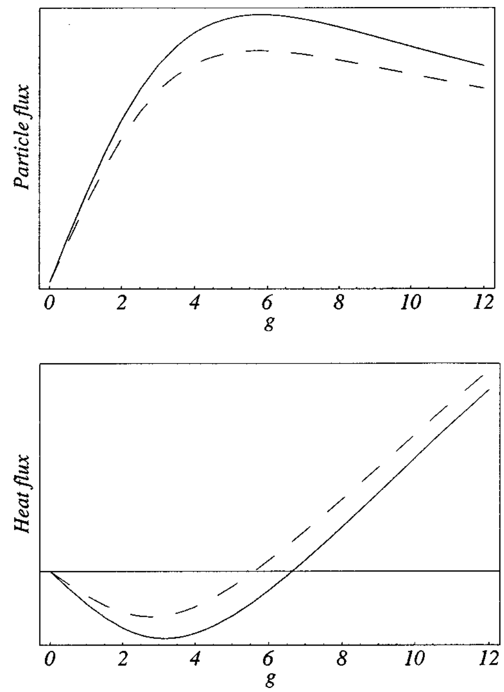

FIG. 4. Neoclassical particle (a) and heat (b) fluxes as functions of the gradient g in START discharge No. 35096, for impurity Mach numbers $M_0^2=0$ (dashed line), and $M_0^2=5$ (solid line). The fluxes are calculated from Eqs. (39) and (40). For the calculation of the heat flux it is assumed that $n_i'/n_i\simeq 2T_i'/T_i$.

gives the following solution in lowest order:

$$n_0=\frac{\gamma}{(1-\langle(1+\gamma b^2)^{-1}\rangle)}\frac{b^2}{1+\gamma b^2},$$

and the neoclassical cross-field particle flux (34) becomes

$$\langle\boldsymbol{\Gamma}^{\rm neo}\cdot\nabla\psi\rangle=-\frac{I\langle p_z\rangle\langle\mathbf{B}\cdot\nabla\theta\rangle\gamma}{e\langle B^2\rangle(1-\langle(1+\gamma b^2)^{-1}\rangle)}\left\langle\frac{\partial M^2/\partial\vartheta}{1+\gamma b^2}\right\rangle. \quad (41)$$

The main conclusion of Ref. 8 was that the neoclassical particle flux is suppressed when $g\gg 1$. It is now clear that this suppression need not be complete, but there can be a residual transport flux governed by the rotation. This transport, which for instance could occur in a steep edge transport barrier, has a number of surprising properties. It can be either inward or outward, and it depends on the geometry of the magnetic field in a nontrivial way.

For instance, consider the case $\gamma\ll 1$. Then the flux (41) becomes

$$\langle\boldsymbol{\Gamma}^{\rm neo}\cdot\nabla\psi\rangle=-\frac{u\langle n_z\rangle\langle\mathbf{B}\cdot\nabla\theta\rangle}{(p_i'/p_i)-(3/2)(T_i'/T_i)}\left\langle b^2\frac{\partial M^2}{\partial\vartheta}\right\rangle.$$

The properties of this expression depends on the concentration of impurities. In a very impure plasma, $\alpha\gg 1$, the quantity u is given by Eq. (14). The flux is then negligibly small as it is inversely proportional to the gradients and thus comparable to that caused by the term n_1, which we neglected in

Eq. (41). In the opposite limit of trace impurities, $\alpha \ll 1$, u is given by Eq. (20), and the flux remains finite at large gradients,

$$\langle \mathbf{\Gamma}^{\mathrm{neo}} \cdot \nabla \psi \rangle = 0.33 \frac{f_c I \langle p_z \rangle}{e \langle B^2 \rangle^2 [(d \ln n_i / d \ln T_i) - (1/2)]} \times \langle B^2 \mathbf{B} \cdot \nabla M^2 \rangle .$$

Here we have used the relation $\langle \mathbf{B} \cdot \nabla \theta \rangle \partial M^2 / \partial \vartheta = \mathbf{B} \cdot \nabla M^2$. Note that this flux is independent of the collision frequency although it is caused by Coulomb collisions. Remarkably, it is proportional to $I = RB_\varphi$ and therefore changes sign if the toroidal field is reversed. If the density profile is at least half as steep as the temperature profile, which is normally the case in the tokamak edge, the flux has the same sign as

$$\langle \mathbf{\Gamma}^{\mathrm{neo}} \cdot \nabla \psi \rangle \propto I \langle B^2 \mathbf{B} \cdot \nabla R^2 \rangle .$$

By recalling the definition of the flux surface average, we see that

$$\langle B^2 \mathbf{B} \cdot \nabla R^2 \rangle = \oint B^2 \frac{\partial R^2}{\partial \theta} d\theta \Big/ \oint \frac{d\theta}{\mathbf{B} \cdot \nabla \theta} = \int_{R^2_{\min}}^{R^2_{\max}} (B^2_{\mathrm{up}} - B^2_{\mathrm{down}}) dR^2 \Big/ \oint \frac{d\theta}{\mathbf{B} \cdot \nabla \theta},$$

where we have chosen θ to increase in the direction of increasing R above the midplane, and vice versa below the midplane. (The precise definition of the angle θ is otherwise still arbitrary.) $B_{\mathrm{up}}(R)$ denotes the field strength on the flux surface in question above the midplane at major radius R, and B_{down} the corresponding field strength below the midplane. In a symmetric equilibrium $B_{\mathrm{up}} = B_{\mathrm{down}}$, but if there is an X-point below the midplane, then normally $B_{\mathrm{up}} > B_{\mathrm{down}}$, and it follows that the particle flux is in the direction of

$$\langle \mathbf{\Gamma}^{\mathrm{neo}} \cdot \nabla \psi \rangle \propto I / \mathbf{B} \cdot \nabla \theta .$$

It is now straightforward to verify that the flux is inward if $\mathbf{B} \times \nabla B$ is downward, and vice versa. Thus, if the ion ∇B-drift is toward the X-point, which is experimentally favorable for attaining the high-confinement H-mode, the neoclassical bulk-ion particle flux is inward, and the impurities (whose flux is opposite to that of the main ions) are prevented from entering the plasma core.

Hinton has earlier identified a different, and perhaps more robust, mechanism which also produces differences between plasmas with opposite signs of the ion drift.[24] Hinton noted that if there is a heat sink at the X-point, the diamagnetic drift is inward if the magnetic drift is toward the X-point, and vice versa. For a given heat flux, the ion temperature gradient will thus be larger in a plasma with $\mathbf{B} \times \nabla B$ toward the X-point than in a plasma where the reverse holds. The mechanism found here is quite different in nature as it does not depend on any localized sources or sinks on the flux surface. Indeed, the asymmetry with regard to the direction of $\mathbf{B} \times \nabla B$ is produced by the plasma itself.

V. SUMMARY

We have explored the effect of rapid toroidal rotation and steep gradients on the neoclassical transport in an impure plasma consisting of collisionless electrons and main ions, and collisional, highly charged impurity ions. Rotation and steep gradients both tend to produce poloidal rearrangement of heavy impurities so that their density varies over the flux surface, something which in neoclassical theory is normally either neglected or ruled out by the orderings. The mechanisms by which rotation and large gradients produce poloidal impurity asymmetry are different, and their effects on the neoclassical transport are also different in nature.

Rapid rotation alone pushes impurity ions to the outside of the flux surface on which they reside, by the centrifugal force. As has been pointed out recently for other collisionality regimes,[15,16] this tends to increase the neoclassical transport. If the impurities are collisional, as in the present paper, the flux enhancement is of the order of the aspect ratio ϵ^{-1}.

A large pressure or temperature gradient of the bulk ions gives rise to a friction force on the impurities, which pushes these toward the inside of the flux surface. The poloidal impurity density variation thus created is small (of order ϵ) at large aspect ratio, but can be very large (comparable to the variation in B^2) at tight aspect ratio and should then be experimentally detectable. Regardless of the aspect ratio, the rearrangement of the impurities greatly reduces their friction with the bulk ions, and as found in Ref. 8 this causes the neoclassical transport fluxes to become *nonlinear* functions of the gradients. When the latter are sufficiently large the neoclassical particle flux is suppressed and the heat flux can be significantly reduced.

However, if the plasma rotates toroidally something remarkable happens in the limit of large gradients. The neoclassical particle flux is then not completely suppressed but instead acquires a number of unusual features. In the particularly simple limit of low impurity concentration, the particle flux becomes independent of the collision frequency (although it is caused by collisions), like the bootstrap current, and is sensitive to the geometry of the magnetic equilibrium. The flux can have either sign and is *inward* if the ion magnetic drift is toward the X-point in a single-null magnetic configuration. The impurities, whose flux is in the opposite direction, are then screened from the plasma core. These fluxes change sign if the toroidal field is reversed.

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