Reply to Comment: "Mode conversion of waves in the ion-cyclotron frequency range in magnetospheric plasmas"

Ye.O. Kazakov
Laboratory for Plasma Physics, LPP-ERM/KMS, TEC Partner, BE-1000 Brussels, Belgium

T. Fülöp
Department of Applied Physics, Chalmers University of Technology, SE-41296 Göteborg, Sweden

(Dated: August 24, 2014)

Previously, we evaluated conditions under which an efficient mode conversion (MC) of fast magnetosonic waves (FW) can occur in planetary magnetospheres and discussed how these results can be used to estimate the concentration of heavy ions [1]. Here, we show that the arguments raised in the preceding Comment [2] do not influence the claims and conclusions of our paper. Furthermore, numerical results reported in the Comment support our previous analytical findings.

a) Kim and Johnson argue that the FW dispersion relation cannot be simplified to a cutoff-resonance pair close to the crossover frequency because of the appearance of a cutoff-resonance-cutoff (CRC) triplet. The MC efficiency for an isolated cutoff-resonance layer and for a CRC triplet [3, 4] are given by the expressions $\mathcal{C}^{(\mathrm{Budden})} = \mathcal{T}(1-\mathcal{T})$ and $\mathcal{C}^{(\mathrm{triplet})} = 4\mathcal{T}(1-\mathcal{T})\sin^2(\Delta\phi/2)$, where $\mathcal{T} = e^{-\pi\eta}$ is the transmission coefficient and η is the tunneling factor. Whereas for the isolated layer, the maximum MC efficiency is 25%, for the CRC triplet it may reach 100% provided: (i) reflected waves have opposite phases, and (ii) the MC layer is semi-transparent $\mathcal{T} \approx 0.5$. The latter condition for having maximized MC is *common* for both models, and paper [1] identified proper conditions for that. If the MC layer is non-transparent – as we prove is the case for the waves with small $k_\parallel$ – almost total reflection at the MC layer occurs and waves cannot be detected by the satellite. When the parallel wave number $k_\parallel$ is close to the critical $k_\parallel^*$, the FW dispersion indeed includes R-cutoff and the Taylor expansion at the resonance allows one to estimate the FW tunneling and the maximum MC efficiency, $\mathcal{C}_{\max} = 4\mathcal{T}(1-\mathcal{T})$. This discussion does not change the conclusions given in [1]: efficient MC can occur for the waves with $k_\parallel \lesssim k_\parallel^*$, having a frequency close to the crossover frequency.

b) The quantity $k_{\parallel,cr}$ introduced in Ref. [2], in general, is different from the critical wave number $k_\parallel^*$ ($k_{\parallel,cr} \geq k_\parallel^*$) derived in [1]. Hence, reporting $k_\parallel/k_{\parallel,cr} < 1$ is not surprising. Here, we confirm the validity of our initial statement that efficient MC requires $k_\parallel/k_\parallel^* \lesssim 1$. In Refs. [2, 5], the maximum MC was computed to occur at $(m_z, X[\mathrm{Na}^+], \omega_S/\omega_{\mathrm{cH}}) = (1, 0.12, 0.2), (2, 0.25, 0.4)$, and $(3, 0.45, 0.6)$, where $m_z = k_\parallel R_M/3.38$ and R_M is Mercury's radius. Substitution of these values into Eq. (8) in Ref. [1] yields $k_\parallel/k_\parallel^* \approx 0.93$ for all three cases. This proves the accuracy of our analytical result.

The frequency f appearing in Eq. (8) is the resonant frequency from the ULF measurements f_{obs} and is attributed to the MC frequency. As stated in Ref. [1], for conditions when MC is efficient, f_{obs} is close, *but somewhat below*, the crossover frequency. Figure 1 shows the sodium concentration as a function of $k_\parallel$, computed for $f_{\mathrm{obs}}/f_{\mathrm{cH}} = 0.5\,\mathrm{Hz}/1.31\,\mathrm{Hz} \approx 0.38$ and fixed resonance location [6]. The colour of points represents $\mathcal{C}_{\max}$. For Mercury, $\mathcal{C}_{\max} = 1$ is reached at $k_\parallel/k_\parallel^* \approx 0.93$ and $X[\mathrm{Na}^+] \approx 23.3\%$. Hence, $\omega_{\mathrm{cr}}/\omega_{\mathrm{cH}} \approx 0.48$, and the ratio of the resonant to the crossover frequency is $p = \omega_S/\omega_{\mathrm{cr}} \approx 0.79$. In general, accounting for $p \lesssim 1$, leads to the modification of the sodium estimate as $X[\mathrm{Na}^+] \approx X_{\min}[\mathrm{Na}^+]/p^2$, where $X_{\min}[\mathrm{Na}^+] \approx (f/f_{\mathrm{cH}})^2$ is the sodium concentration corresponding to the crossover condition at the observation point for the given frequency. These values are fully consistent with Ref. [5]. Note that for the plasma conditions near the geostationary Earth orbit, the frequency correction is much smaller, $p \approx 0.97$.

The $m_z = 1$ case considered in Ref. [2] (for which $\omega_S/\omega_{\mathrm{cr}} \approx 0.56$) corresponds to $f/f_{\mathrm{cH}} = 0.2$. Such a frequency ratio is not experimentally relevant to the best of our knowledge: the Mariner 10 and MESSENGER spacecrafts detected $f/f_{\mathrm{cH}} \simeq 0.4$ and $f/f_{\mathrm{cH}} \simeq 0.7 - 0.8$, respectively [6, 7]. Therefore, for the measured resonant ULF waves at Mercury $\omega_S/\omega_{\mathrm{cr}}$ is ∼0.8 or higher, supporting the conclusions of Ref. [1].

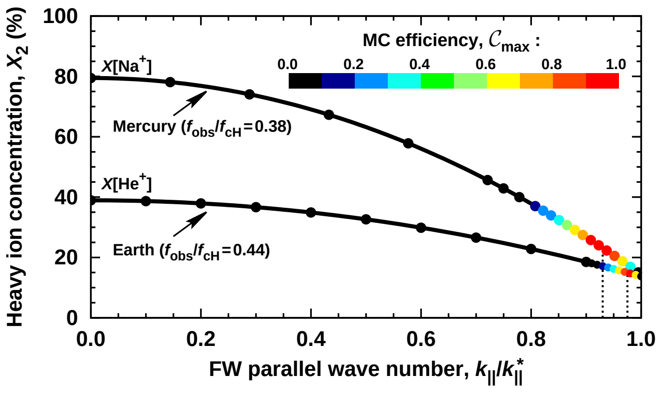

FIG. 1. Heavy ion concentration vs. $k_\parallel$ for the fixed resonance location.

[1] Ye.O. Kazakov, and T. Fülöp, Phys. Rev. Lett. **111**, 125002 (2013).
[2] E.-H. Kim and J.R. Johnson, preceding Comment, Phys. Rev. Lett. **113**, 089501 (2014).
[3] V. Fuchs, A.K. Ram, S.D. Schultz, A. Bers, and C.N. Lashmore-Davies, Phys. Plasmas **2**, 1637 (1995).
[4] A.K. Ram, A. Bers, S.D. Schultz, and V. Fuchs, Phys. Plasmas **3**, 1976 (1996).
[5] E.-H. Kim, J.R. Johnson, and K.-D. Lee, Geophys. Res. Lett. **38**, L16111 (2011).
[6] C.T. Russell, Geophys. Res. Lett. **16**, 1253 (1989).
[7] S.A. Boardsen, J.A. Slavin, B.J. Anderson et al., J. Geophys. Res. **117**, A00M05 (2012).